# ESTIMATION OF STATES IN PHOTOSYNTHETIC SYSTEMS VIA *CHAINED OBSERVERS*: DESIGN FOR A TERTIARY WASTEWATER TREATMENT BY USING *Spirulina maxima* ON PHOTOBIOREACTOR

## ESTIMACIÓN DE ESTADOS EN SISTEMAS FOTOSINTÉTICOS MEDIANTE OBSERVADORES ENCADENADOS: DISEÑO PARA UN TRATAMIENTO TERCIARIO DE AGUAS RESIDUALES EN CULTIVO DE *Spirulina maxima* EN FOTOBIORREACTORES

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## Abstract

The cultivation of microalgae has currently shown a growth in interest because it has shown significant contributions to the production of energy, high quality food and as agents for tertiary treatment of wastewater. Microalgae culture are carried out on photobioreactors, which are often very complex. Therefore, the automatic control of these devices can be very difficult and the lack of in-line sensors makes it difficult to control them. For this purpose, state observers have proven to be an excellent tool for measuring variables that cannot be measured online. In this paper a theoretical design of a robust linear type observer, the so-called *Chained Observer* is proposed. Stability is demonstrated through Lyapunov functions and an observer is didactically designed for the study of tertiary wastewater treatment by demonstrating that under certain conditions, the *Chained Observer* presents better yields than even more sophisticated observers. A simple technique for multi-disciplinary researchers is presented.

*Keywords*: *Spirulina maxima*, observers, photobioreactors, Lyapunov.

## Resumen

En la actualidad, el cultivo de microalgas ha sido de gran interés por sus aportaciones en la producción de energía, desarrollo de alimentos de alta calidad y uso como agentes para el tratamiento terciario de aguas residuales. El cultivo de microalgas se realiza en fotobiorreactores, a menudo con dinámicas muy complejas. Por lo tanto, el empleo de técnicas de control automático aplicadas en estos dispositivos puede resultar muy difícil, ya que un problema común es la falta de sensores en línea para su control. Con este propósito, los observadores de estado han demostrado ser una excelente herramienta para medir variables que no pueden ser medidas en línea por algún sensor. En este trabajo se propone un diseño teórico de un observador robusto de tipo lineal denominado *Chained Observer* (observador encadenado). El estudio de estabilidad se demuestra a través del uso de funciones de Lyapunov. Un observador encadenado es diseñado de manera didáctica para el estudio del tratamiento terciario de aguas residuales demostrando que bajo ciertas condiciones, el *Chained Observer* presenta mejores rendimientos que los observadores aún más sofisticados. En este trabajo se presenta una técnica sencilla de entender para investigadores multidisciplinarios.

*Palabras clave*: *Spirulina maxima*, observadores, fotobiorreactores, Lyapunov.

## 1 Introduction

The human population has grown at a very high rate since the middle of the last century as it has never been seen in history. The demand for energy sources, pharmaceuticals and the production of food products also has a very high demand rate, thus modern science must put all its efforts into improving the processes of obtaining them Myers (2002). The vast majority of these resources are obtained through complex processes called bioprocesses.

Bioprocesses play an effective role in the production of value-added products in the pharmaceutical, food and energy industries. Therefore, the study and improvement of these processes is an important topic for current research. A very special group of these bioprocesses are carried out through photosynthetic processes Zhang *et al.* (2018). The natural process that allows the production of biomass using only sunlight as the energy source and carbon dioxide is photosynthesis. Photosynthesis is the photochemical process carried out by special kind of cells to produce the inverse effect of mineralizing, i.e. converting inorganic matter into organic matter, by fixing the energy from the sun (Cañedo & Lizárraga, 2016). The origin of life on earth began with a reduced atmosphere. The atmosphere became oxidizing through the production of oxygen from photosynthesis carried out by the proliferation of cyanobacteria from 2,400 to 3,200 million years ago. With oxygen production, the ozone layer was developed in the stratosphere which filters part of the UV-B (280-315 nm) radiation, helping the development of all sorts of living organisms on the planet (del Rio-Chanona *et al.*, 2018; Ooms *et al.*, 2017; Yun & Park, 2003).

Photobiochemical crops and microalgae fermentations have recently received much attention and scientific study due to the production of high nutritional quality foods (e. g. cyanobacteria: *Spirulina maxima*) and pharmaceutical production of high value-added by-products, as well as $CO_2$ fixation (thereby helping to mitigate the greenhouse effect) and renewable energy production. This situation can be related to the high photosynthetic performance and the main and cleanest source of energy: solar energy. Compared to other living organisms such as terrestrial plants whose growth is limited by the availability of $CO_2$, massive cultivation of microalgae leads to a high potential for algae biomass production of several tens of tons per hectare per year (Cañedo & Lizárraga, 2016).

Microalgae biological systems are the most efficient systems known for capturing sunlight in nature. They produce proteins from inorganic sources through the photosynthetic process. Millions of years of evolution have helped to maximize the optimization of energy efficiency in aqueous photosynthesis. This could represent an immediate solution to meet the high demand for energy and food products. Fixing of energy from inexhaustible primary sources could contribute directly to meet the food needs of the growing demand of the above-mentioned exponentially growing of human population (del Rio-

Chanona *et al.*, 2018; Ooms *et al.*, 2017; Yun & Park, 2003). Microalgae are usually grown in devices called phobioreactors. A photobioreactor is a special type of bioreactor where biochemical and photochemical reactions are carried out, resulting in the production of microalgae biomass. Bioreactors are controlled devices whose purpose is to maximize the production of biochemical products through the control of variables such as flow, volume, temperature, pH, dissolved oxygen, among others (Junker & Wang, 2006).

Growing microalgae on a large scale can be done in open systems (lakes or ponds) and in closed controlled systems called photobioreactors. Outdoor pond systems are cheaper to build. They only require a ditch or pond but they are usually difficult to control. In an open pond culture, perishability performance can be easily disturbed by natural distortions such as: microbial contamination, temperature changes, light, pH, availability of carbon dioxide, among others. In contrast, photobioreactors are closed and controlled systems that allow the optimal operation, minimize pollution, achieve maximum production and allow the use of axenic algae and monoculture cultivation. Therefore the cultivation of microalgae in photobioreactors has many advantages over open systems (Singh & Sharma, 2012).

For a high performance, the design requirements and conditions that a photobioreactor must meet are the following:

1. Reactor must provide the culture conditions of several types of microalgae.

2. The reactor design should ensure uniform controlled illumination of the irradiation zone and an excellent distribution of photons from the natural or artificial light source.

3. Fast mass transfer of $CO_2$ and $O_2$.

4. The photobioreactor sometimes works under intense foaming conditions, such as reactors with high mass transfer rates.

5. The reactor must have a minimum lighting section to ensure the light-dark cycle necessary for the photosynthetic metabolic process in order to get an extra control variable on cell stress. It is known that many microalgae can be induced to stress in controlled cycles of light and darkness. Such stress is usually used for the production of some kind of high value-added metabolite (Cañedo & Lizárraga, 2016).

For an excellent operation of the different types of photobioreactor devices, precise and reliable control of monitored variable must be guaranteed. But controlling biological systems is a challenge for modern control theory. In more recent decades, the automation of the cultivation of microorganisms in bioreactors has been investigated in a wide range of ways. The control of such processes can be complex and costly, since they are highly non-linear systems from their phenomenological description and mathematical modeling (Bernard, 2011). In addition, sensors to measure relevant crop variables (biomass and metabolites of interest) are often very expensive, non-existent in the commercial market or do not meet the required technical standards, such as response time, accuracy and reliability. It is important to remember that a sensor is a mechanism or device that converts an actual physical variable and characterizes it to a digital or analogical electrical variable in order to be processed in an automated control system (Lyubenova *et al.*, 2013). Modern control theory offers analytical and numerical tools to solve the task of estimating variables in which they cannot be measured, the sensors are expensive or the measurement variable is inaccessible. Such tools are often referred to as observers or state estimators. Biological systems, especially those made in bioreactors or photobioreactors, are the ideal processes for the application of these techniques for estimating bioreactor variables. An observer is a dynamic system which tasks online estimation of states that cannot be measured at the output of a systems (Lyubenova *et al.*, 2013; Rodríguez-Mata *et al.*, 2011; A. Rodriguez-Mata *et al.*, 2015; Celikovsky *et al.*, 2015; Farza, 2014; Gauthier *et al.*, 1992). The output variables are the variables that can be measured in a process through a sensor. Some examples of such variables in a typical chemical reactors are the concentration of reagents or products, temperature, pressure, among others. The control variables or inputs are the variables that modify the dynamics of a system. These variables are usually managed through actuators such as valves, circuits, resistances, among others.

The so-called Luenberger observer is the most widely used asymptotic observer, especially in linear systems. Extension to non-linear systems has been studied in several versions of observers, such as those shown in Doko (2017) and Liu *et al.* (2017). Within these non-linear observers, high gain non-linear observers stand out, whose structure is not fundamentally dependent on the input. Under the hypotheses of a perfect knowledge of plant parameters and the absence of unknown disturbances, these observers have shown excellent performances in non-linear systems and used in some types of simple bioreactor models (Gauthier *et al.*, 1992). There are limited studies about the online estimation of the states of variables in photobioreactors using state observation techniques, due to the high complexity of these type of systems, but some works in which it has been demonstrated demonstrating that the use of non-complex techniques such as Luenberger observers allow for exclusive results when dealing with estimation problems in this type of system (Benavides *et al.*, 2015). In this paper, the use of *Chained Observers* state estimators are proposed for the estimation of variables in photobioreactors. A didactic exposition of basic concepts of modern control on the use of state observers is presented in such a way that a multi-disciplinary reader (non-automatic control specialists) can understand such control theory. The use and develop of control techniques used for the solution of marked problems in the estimation of variables in photobioreactors are also discussed. The basic concepts and definitions used for the development of observers and the mathematical models used for the description of photobioreactors are presented. Finally, numerical simulation studies are proposed.

## 2 Materials and methods

### 2.1 Preliminary observer theory

Estimating the status of variables that are not measurable or present in a process (also called plant) could be possible by using observer theory. A system is suitable to be observable, if it fulfills certain conditions as a condition of observability.

**Definition 2.1.** *Observability of system*
A state of a system o process can be reconstructed through the input, output and mathematical model information of a dynamic system. This system may be represented by nonlinear systems:

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$
$$y_{nl} = h(x(t))$$

in this work, it is proposed that $x(t)$ is the $n-$dimensional state vector, $u(t)$ is the $m-$dimensional

output vector and $y_{nl}(t)$ is the $p-$dimensional output vector. It is assumed that $f(x(t), h(t))$ are soft vector fields, ensuring the existence and uniqueness of the (1) solution such that $f(x(t), h(t))$ are Lipschitz functions and $u(t)$ is a soft and bounded function.

*Remark.* A particular case of the non-linear system (1) is when it is represented by a linear plant or the so-called linearized system, which therefore complies with the principles of association and overlap. This can be represented by the following:

$$\dot{x}_l(t) = Ax_l(t) + Bu(t) \tag{2}$$
$$y_l = Cx_l(t)$$

where $\dot{x}_l(t)$ describes the dynamics of the system, $x_l(t)$ describes the linear vector states, the term $A \in \mathbb{R}^{nxn}$ is a linearly independent square matrix originated from the Jacobian matrix calculation at a point of equilibrium of states , $B \in \mathbb{R}^{nx1}$ is the input matrix similarly originated from the Jacobian matrix calculation at a point of equilibrium of the variable $u(t)$, $C \in \mathbb{R}^{nx1}$ is the output matrix for $n$ as the system dimension for single input single output (SISO) systems and output vector and $y_l(t)$ is the $p-$dimensional output vector.which are the systems to deal with in this paper.

Soft set theory is a new mathematical tool developed to deal with uncertainties in modelling problems with incomplete information and is used in engineering, physics, computer science, economics, social sciences and medical sciences. This theory does not require the specification of parameters. It gives an approximate description of an object as its starting point. The application of this theory is now a common approach in several disciplines and real life problems (Hussain, 2015).

**Definition 2.2.** *Observer of state* The following auxiliary system is an observer for the system (*1*):

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) - \psi(x(t), \hat{x}(t), u(t))e(t) \tag{3}$$
$$\hat{y}(t) = h(x(t))$$
$$e(t) = \hat{x}(t) - x(t) \tag{4}$$

*such that:*
1.- $\hat{x}(0) = x(0) \rightarrow \hat{x}(t) = x(t)$
2.- $\|e(t)\| = \|\hat{x}(t) - x(t)\| \rightarrow 0 \ \forall \ t \rightarrow \infty$.

In the above definition, the $\psi(\cdot) = \psi(x(t), \hat{x}(t), u(t))$ is a correction function that can be linear or non-linear and represents the feedback used to stabilize the estimation error $e = \hat{x}(t) - x(t)$).

The objective of the $\psi(\cdot)$ correction function is to obtain asymptotic (for $k_1 > 0$ and $k_2 > 0$) of the estimation error in order to guarantee that the estimated variable is the non-measurable variable (Liu *et al.*, 2017).

$$\|e(t)\| \leq k_1 e(t)^{-k_2 t} \ \forall \ t > t_0. \tag{5}$$

It is often difficult to achieve an exponential stability criterion, as in many systems the uncertainties that are not constant over time make it difficult. For this reason, more practical stability criteria have been defined in the literature see to Khalil (2002a), as shown below:

**Definition 2.3.** *Ultimated Bounded Stability (U.B.S)* The solution of non-linear differential equation $e(t)$ are U.B.S. if there exists a positive constant $c$, independent of $t_0 \geq 0$ and every $a \in (0, c)$, there $B = B(a) > 0$ independent of $t_0$ such that the solution will tend to be inside a ball of attraction or convergence B:

$$\|e(t_o)\| \leq a \Rightarrow \|e(t)\| \leq B, \forall t \geq t_0 \tag{6}$$

On the other hand in the vast majority of results in the literature the main condition that a system of non-linear differential equations as (1) must fulfill to be able to use the observer as (3) for the estimation of states is shown in the following definition.

**Definition 2.4.** *Uniform Autonomous Observability (U.O.)*
The system (1) is U.O. if it exists diffeomorphism ($F_x : \mathbb{R}^n \rightarrow D_x \mathbb{R}^n$) for all input $u(t)$ and any initial condition as follows (more details of this see to Gauthier *et al.* (1992)):

$$\begin{cases} F_x : \mathbb{R}^n \rightarrow D_x \mathbb{R}^n \\ \\ x(t) \rightarrow \begin{pmatrix} h(x) \\ L_f h(x) \\ \cdot \\ \cdot \\ L_f^{n-1} h(x) \end{pmatrix} \end{cases} \tag{7}$$

where $L_{fu}$ is Lie derivation operator (derived under a system trajectory) for output-state with $L_{fu}\varphi = \frac{\partial \varphi}{\partial x} f(x, u)$.

*Remark.* A diffeomorphic change of coordinates (7) on a invariant linear system as shown in equation (2) can be designed using the Lie derivation operator as follows:

$$f(x,u,t) = Ax_l(t) \qquad (8)$$
$$h(x,u,t) = Cx_l(t)$$

Then:

$$L_f h(x) = CA(x_l) \qquad (9)$$
$$L_f^{n-1} h(x) = CA^{n-1}(x_l)$$
$$L_f^n h(x) = CA^n(x_l)$$

Such that:

$$\begin{pmatrix} h(x) \\ L_f h(x) \\ \cdot \\ \cdot \\ L_f^{n-1} h(x) \end{pmatrix} \rightarrow \begin{pmatrix} C \\ CA \\ \cdot \\ \cdot \\ CA^{n-1} \end{pmatrix} = O \qquad (10)$$

where $O$ is Kalman's classic observability matrix (Hussain, 2015). In order to be the (3) to be observable it is necessary that the range of the matrix be full (linearly independent).

## 2.2 Linear Observer

Let linear observer be as in Liu *et al.* (2017):

$$\dot{\hat{x}}_l(t) = A\hat{x}_l + Bu + \psi(\cdot) \qquad (11)$$
$$\hat{y}_l = C\hat{x}_l$$
$$\psi(\cdot) = -KCe_l \qquad (12)$$
$$e_l = (\hat{x}_l - x_l)$$

where vector estimated states $\hat{x}_l$ of the above linear observer, $K$ is an output gain matrix and $C$ is the system output matrix defined from the observability analysis (see remark 1). By obtaining dynamic error between (2) and (11) it is obtained:

$$\dot{e}_l(t) = A\hat{x} - Ax - KCe_l$$
$$\dot{e}_l(t) = A_c e$$
$$e_l(t) = \exp(A_c t)$$
$$A_c = A - KC$$

where $e_l(t)$ is defined as estimation linear states error. If the whole set of eigenvalues ($\lambda_i$) defined as output feedback system spectrum ($\sigma_{(\lambda_i)[A-KC]}$) of the matrix $A_c$ has negative real part (Hurwitz condition), then the error will converge to zero when time tends to infinity, such that:

$$\sigma_{(\lambda_i)^{[A-KC]}} = \{\lambda_1, \lambda_2, \ldots \lambda_n\} \in \mathbb{R}^-$$

Then:

$$\|e_l(t)\| = \|\exp(A_c t)\| \le k_1 \exp(-k_2 t) \text{ for } k_1, k_2 > 0 \qquad (13)$$

thereafter:

$$\|e(t)\| \rightarrow 0 \text{ when } t \rightarrow \infty$$

A non-linear system can be linearized and therefore treated as a linear system in a certain region of the function domain. A particular case is the photobioreactor which is a system where its mathematical model is usually highly non-linear but, under certain operating conditions, a robust linear observer can estimate variables via output feedback even deliver superior results in comparison to highly complex non-linear techniques for specific case studies as. This can be of a great interest to easy design state estimators for multidisciplinary users in diverse microalgae cultures and this will be presented in the following chapters of this paper.

## 2.3 Preliminaries of mathematical model

This part describes dynamics of the microalgae growth process in a photobioreactor culture in terms of the Monod model. This formulation is one of the most widely used in mathematical models for biochemical problems. Although it is a non-linear model, it represents sufficient information in the system without being too complex for using the analytical tools of automatic control. With the Monod model, it is possible to represent any type of photobioreactor: open lagoon, tubular, among others, independently of the type of cultivation (batch, fedbatch or continuous).

### 2.3.1 A photobioreactor model

A photobioreactor is a biological active and regulated system where photochemical and biochemical processes are carried out using living organisms. This system is sensitive to changes under ambient conditions. Therefore, the optimal conditions of the microalgae culture must be ensured, such as pH,

light intensity, temperature, oxygen concentration and carbon dioxide (Bernard, 2011; Zhang *et al.*, 2018). The following model of a photobioreactor in batch phase is presented:

$$\frac{dx}{dt} = r(s, pH, T, v.V, O_2, CO_2, C_{Hl}) \cdot x \qquad (14)$$
$$\frac{ds}{dt} = -m_3^{-1}\mu(s, pH, T, v.V, O_2, CO_2, CHl) \cdot x$$
$$x(t_0) = \hat{x}_l \quad s(t_0) = s_0 \quad t \geq 0$$
$$\mu(\cdot) = \mu(s, pH, T, v.V, O_2, CO_2, C_{Hl})$$

where $s$ is the substrate ($mg/L$), $x$ is a concentration of biomass ($mg/L$), $T$ the temperature of the bioreactor ($K$), $v$ the agitation rate (RPM), $V$ the volume of the reactor ($L$), $m_3$ is stoichiometric constant (constant of digestion), $O_2$, $CO_2$, $C_{Hl}$, oxygen concentration, carbon dioxide and chlorophyll respectively ($mg/L$). The $r(\cdot) = r(s, pH, T, v.V, O_2, CO_2, C_{Hl})$ function is a unwieldy non-linear function that represents the complex growth rate of the photosynthetic microorganism with frequency units ($hr^{-1}$) . This representation is usually very complicated, hence it is necessary to make assumptions to develop a mathematical model. This is shown in the following condition.

*Assumption* 1. The growth rate $r(\cdot)$ may be the result of the addition the following terms:

$$r(\cdot) = \mu(s) + \beta(\cdot) \qquad (15)$$
$$\beta(\cdot) = \beta(s, pH, T, v.V, O_2, CO_2, C_{Hl})$$
$$\mu(s) = \frac{m_1 s}{m_2 + s}$$

Where $\mu(s)$ is the growth rate expressed in Monod terms and $\beta(\cdot)$ function is a bounded and unknown function, such that:

**1.-**The parameters $m_1 = a_1 \pm d_1$ ($hr^{-1}$) and $m_2 = a_2 \pm d_2$ ($mg/L$) are Monod's kinetic parameters ($a_1$ maximun exponential rate and $a_2$ saturation constant) and $d_i$ are the maximum statistical averages of possible disturbance or changes of these parameters. Both may be variable over time due to the high sensitivity that microalgaes can show in the metabolism of the substrate ($s$).

**2.-**The $\beta(\cdot)$ function ($hr^{-1}$) is a Lipschitz function in the time, $|\beta(\cdot)| < k_1 t \; \forall \; k_1 > 0$, therefore $\beta(\cdot) \in C^\infty$.

*Remark.* When the Monod parameters $m_1$ and $m_2$ do not change in time, they are the nominal parameters of the systems.

## 2.3.2 Continuous bioreactor model

General balance of matter for the operation of a continous bioreactor is required, such that the quantities associated to the input and output flows must be included in each incremental change expression.

$$\frac{dx}{dt} = \mu(s)x - u(t)x + \beta(\cdot)x \qquad (16)$$
$$\frac{ds}{dt} = -m_3^{-1}\mu(s)x - \beta(\cdot)x + u(t)(a_4 - s)$$
$$\mu(s) = \frac{m_1 s}{m_2 + s} \qquad (17)$$

The above system of differential equations (16) will be considered as the working model in this paper. The dilution ratio of the feed $u(t) = F/V$ ($hr^{-1}$) is considered as the flow rate ($F$) quotient of input and output over the volume of the photobioreactor ($V$) and represents the control input of the system and $a_4$ is feeding input concentration substrate ($mg/L$). In the literature, it is usual for this type of system to use the main state variable for the design of control laws (see for example: Pimentel *et al.*, 2015; Dewasme *et al.*, 2011). In this direction, a robust control law based on variable biomass $x(t)$ can be obtained on the control variable $u_{robust}(t)$ for the cancellation of uncertainties $\beta(\cdot)$, through natural bi-linearity properties of the system (16).

$$\frac{dx}{dt} = \mu(s)x - u_{robust}(t)x + \beta(\cdot) \qquad (18)$$
$$u_{robust}(t) = u(t) - \hat{\beta}(\cdot)$$

With a good estimation of the unknown function $\hat{\beta}(\cdot) \approx \beta(\cdot)$, this disturbance can be rejected and the Monod's typical behavior (18) is presented due to absence of disturbances. This situation would help greatly in order to use optimal control open-loop, using dilution rate to optimize productivity biomass. The Optimum productivity of a microalgae culture (without disturbance effect) is defined as product of the dilution ratio by concentration of the microorganism in a stationary state (Dominguez-Bocanegra A., 2002):

$$P = u(t)x_{est}$$

Where $x_{est}$ (mg/L) is biomass stationary equilibrium point. Based on this, an optimal dilution rate is proposed to maximize biomass productivity.

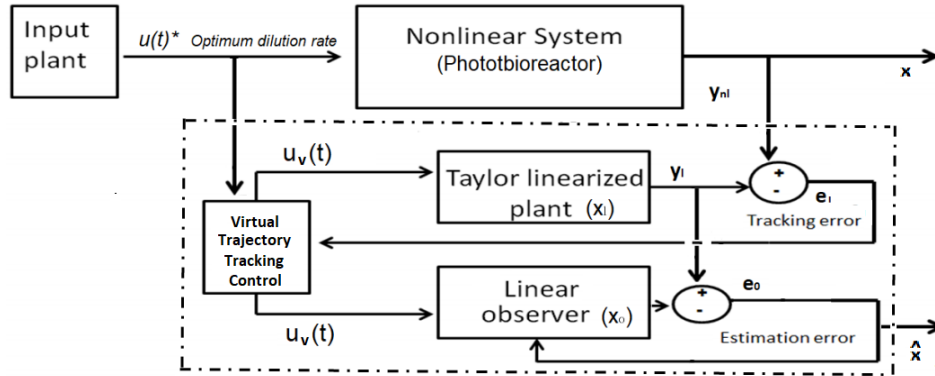$$u^{opt}(t) = a_1(1 - \sqrt{\frac{a_2}{a_2 + a_4}}) \qquad (19)$$

Fig. 1. Schematic diagram of the *Chained Observer*.

### 2.3.3 A chained observer

This chapter of this paper deals with the design of a robust type of linear observer for the particular case of the nonlinear system shown (1) for $\mathbb{R}^2$. The dynamics of a linear system (11) can be modified in such a way that it can follow the dynamics of another system, i.e. design a *Trajectory Tracking Control (T.T.C.)*. Based on this, a virtual *T.T.C.* ($u_v(t)$) can be designed such that a linearized system (11) follows dynamics of original non-linear system (1), even in presence of disturbances or moving away from equilibrium point. For this virtually controlled system can be design a state observer, resulting finally in sum of 2 linear systems interconnected, this will be treated below.

Let a particular case of the system (1) be in the following equation for $\mathbb{R}^2$:

$$\dot{x} = f(x) + g(x)(u(t) + \beta(\cdot)) \qquad (20)$$
$$y_{nl} = Cx$$

*Assumption* 2. The previous system (20) contains soft and limited functions $f(x), g(x) \in \mathbb{R}^2$ such that:
1.- $\lim_{t \to \inf} f(x) = f_{est}$
2.- $\lim_{t \to \inf} g(x) = g_{est}$
3.- $f_{est}$ , $g_{est} \in \mathbb{R}^2$ are vector constants.

In some systems, assumption 2 is fulfilled in certain systems (Rodriguez-Mata *et al.*, 2015; Bernard, 2011), such as in the case of chemical, biochemical and biological systems and for all types of systems that converge at the point of natural equilibrium ($x_{eq}$). In this sense, the system (20) can be represented as a linear system (2) using the Taylor approach (Jacabian matrix) in an equilibrium point ($x_{est}$) in addition to aforementioned virtual *T.T.C.*

Let the following linear system be:

$$\dot{x}_l = Ax_l + bu_v(t) \qquad (21)$$
$$y_l = Cx_l$$

where $x_l \in \mathbb{R}^2$ is state vector, $A \in \mathbb{R}^{2x2}$ is a Jacobian matrix respect to state matrix of non-linear system (20), $b \in \mathbb{R}^2$ is a Jacobian matrix respect to input of non-linear system (20), $y_l$ is the output and $C$ is the otoput matrix. The following linear observer is proposed for linearized system (21):

$$\dot{\hat{x}}_l = A\hat{x}_l + bu_v(t) + Ke_0 \qquad (22)$$
$$y_l = C\hat{x}_l$$
$$e_o = \hat{x}_l - x_l$$

here $\hat{x}_l \in \mathbb{R}^2$ is vector of estimated states, $K$ is a gains Hurwitz matrix and $e_o$ is observer error. An important element to use is defined below.

**Definition 2.5.** *Total Output Injection Error*
When in a system (20) it is not possible to access all measurements from all states, it is said that *Total Output Injection Error* ($e_T$) is difference between measurable states ($y_{nl} = Cx$, via sensors) and observer (22).

$$e_T = Cx_l - y_{nl} = C(x_l - x) \qquad (23)$$

*Remark.* In a system of type (20) where $x \in \mathbb{R}^2$. For instance if it has an output matrix $C = [1 \ 0]$, $x = [x_1 \ x_2]^T$ and a linear observer as (21). Then it is obtained:

$$e_T = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \hat{x}_1 - x_1. \qquad (24)$$

The main problem in this section is the possibility of estimating $x$ of (20) under the presence of uncertain disturbance terms $\beta(\cdot)$. Then, this disturbance function is estimated and rejected via $u_v(t)$. For this purpose, interconnection of a virtual *T.T.P.* control $(u_v(t))$, a linearized system (21) and a linearized system observer (22) is proposed. This interconnection is called the *Chained Observer*. This result is shown in following system:

$$\dot{x}_l = Ax_l + bu_v(t) \qquad (25)$$
$$\dot{\hat{x}}_l = A\hat{x}_l + bu_v(t) + Ke_0$$
$$u_v(t) = u(t) - \hat{\beta}(\cdot)$$
$$\dot{\hat{\beta}}(\cdot) = \kappa_c \dot{\epsilon}(t)$$

where:

$$\epsilon = [e_T \int e_T dt]^T$$
$$\kappa_c = [k_p \ k_i]^T$$

The previous system (25) is chained via $u_v(t)$ which is a form of PI control with saturation $u_v(t) = u(t) + \hat{\beta}(\cdot)$ (in this work $u(t) = u^{opt}(t)$ within meaning (19)). Estimation of uncertainty or disturbance $\hat{\beta}(\cdot)$ is based on an output feedback and the positive defined gains $k_p, k_i > 0$. The stability of the state global error $(e = x - x_l)$ is achieved using following main theorem by $\mathbb{R}^2$ system.

*Theorem* 2.1. *Chained Observer*

Let system (25) be a *Chained Observer* of the non-linear system (20). Then global error $e = x - x_l$ presents Ultimated Bounded Stability only if following conditions are fulfilled:

1. The pair $(A, C)$ of (21) is completely observable in the sense of (10).

2. The $\beta(\cdot) \in C^\infty$.

3. The $K$ is a Hurwitz matrix.

4. The $k_p, k_i$ are proportional and integral gain, such that:
   $\Gamma = \begin{pmatrix} bk_p & b \\ k_i & 0 \end{pmatrix}$ is a Hurwitz matrix.

5. $f(x), g(x), \beta(\cdot) \in C^\infty$.

*Proof.* The following will prove stability of the estimation error of the chained observer. This situation is demonstrated via Lyapunov functions. Since two linear systems are interconnected (25), it is possible to use principle of separation. The proposed proof methodology is divided into two parts: virtual control stability proof and robust observation error.

**a).-** It is proposed an order of virtual tracking control law stage( $u_v(t)$ of (25). Let $\dot{e}_T$ be and replacing differents dynsmics (20) and (21) :

$$\dot{e}_T = C\dot{x} - C\dot{x}_l$$
$$\dot{e}_T = C(f(x) + g(x)u(t) + g(x)\beta(\cdot) - Ax_l - bu_v(t))$$

In the same sense , in this paper it is proposed that $C\Lambda_1(\cdot) = f(x) + g(x)u(t) + g(x)\beta(\cdot) - Ax_l$, since functions $f(x), g(x), \beta(\cdot) \in C^\infty$ are soft functions. The virtual control $u_v(t)$ shown (25) is replaced. Therefore:

$$\dot{e}_T = C\Lambda_1(\cdot) - Cbu_v(t)$$
$$u_v(t) = u(t) - \hat{\beta}(\cdot)$$
$$\dot{\hat{\beta}}(\cdot) = \kappa_c^T \dot{\epsilon}(t)$$
$$\epsilon = [e_T \int e_T dt]^T$$
$$\kappa_c = [k_p \ k_i]^T$$

reducing:

$$\dot{e}_T = C\Lambda(\cdot) - Cb(-k_p e_T - k_i \int e_T dt)$$

Then:

$$\dot{e}_T = C\Lambda(\cdot) - Cb(-k_p e - w)$$
$$\dot{w} = -k_i e$$

The *General Tracking Error Function* $(E = [e_T \ w]^T)$ is used and Replacing coordinate change on above expression, it is obtained:

$$\dot{E} = \Gamma E + C\Lambda_T(\cdot) \qquad (26)$$
$$\Gamma = \begin{pmatrix} b_1 k_p & b_1 \\ ki & 0 \end{pmatrix}$$
$$\Lambda_T(\cdot) = \begin{pmatrix} C\Lambda(\cdot) \\ 0 \end{pmatrix}$$

where $\Gamma$ is matrix of distribution is Hurwitz by construction.

The following function is proposed as a candidate of Lyapunov and its derivative lowers the trajectories of the (26) :

$$V = 0.5 E P_1 E^T$$

where $P_1$ is a no negative symmetric matrix, $\dot{V} = \dot{E} P_1 E^T + E P_1 \dot{E}^T$ with $\dot{E}^T = E P_1^T + \Lambda_T^T(\cdot)$ and $\dot{E} = P_1 E + \Lambda_T(\cdot)$. The following is obtained:

$$\dot{V} = E^T(\Gamma^T P_1 + \Gamma P_1)E + 2\Lambda_T(\cdot)P_1 E$$

Since $\Gamma$ is Hurwitz, then matricial Lyapunov equation is obtained $E^T(\Gamma^T P_1 + \Gamma P_1)E = -Q_1$, bounded due to Liszchtz condition of $\|2\Lambda_T(\cdot)P_1 E\| \leq 2h\lambda_{\max}(P_1) = b$ for any $\|\Lambda_T(\cdot)\| < h$. Then,it is possible to limit in same sense of A. Rodriguez-Mata *et al.* (2015):

$$\dot{V} \leq -\lambda_{\max}(Q_1)\|E\|^2 + b\|E\|$$
$$\dot{V} \leq -\lambda_{\max}(Q_1)\|E\|\left(\|E\| - \frac{b}{\lambda_{\max}(Q_1)}\right)$$

For values of $b$ smaller than $\lambda_{\max}(Q_1)$ where $Q_1$ is a solution matrix matrix of the Lyapunov matrix equation.
Virtual tracking control maintains the error $e_T$ with a ultimated bounded stability with a ratio of convergence ball as a function of $\lambda_{\max}(Q_1)$ in the sense Khalil (2002b).

**b).-** The test stage of the observer between the linearized system and Luenberger linear observer part (22). Let linear system and observer system be:

$$\dot{x}_l = A x_l + b u_{robust}(t) \qquad (27)$$
$$\dot{\hat{x}}_0 = A\hat{x}_l + b u_{robust}(t) + K e_0$$

The estimation of states error is proposed: $e_o = x_l - \hat{x}_l$. Through the dynamics of this error, $\dot{e}_o = \dot{x}_l - \dot{\hat{x}}_0$ is obtained. The observer and linearized plant are replaced such that:

$$\dot{e}_o = A x_l - A\hat{x}_l - K e_0 \qquad (28)$$
$$\dot{e}_o = A_K e_0$$
$$A_K = A - K$$

with $V(t) = -0.5 e_0^T P_2 e_0$ and using the derivative with respect to the trajectories of $V(t)$, the following equations are obtained:

$$\dot{V}(t) = -\dot{e}_0^T P_2 e_0 - e_0^T P \dot{e}_0$$
$$\dot{V}(t) = -e_0^T(A_K^T P_2 + P_2 A_K)e_0$$

If the K matrix is chosen properly such that $A_K$ is Hurwitz matrix $A_K^T P_2 + P_2 A_K = -Q_2$, the error $e_o$ is asynthically stable with function of $Q_2$ solution of the algebraic matrix of Lyapunov.

$$\dot{V}(t) = -\lambda_{\max}(Q_2)\|e_0\|^2$$

$\square$

The stability of the chained observer has been demonstrated. By using the principle of separation in the chained observer, it can be argued that the system is total stable, from a practical stability point of view (Khalil, 2002b). Therefore unavailable variable of the non-linear system is estimated. The following techniques will be used in the estimation of states in models of photobioreactors.

## 2.4 Chained observer for a photobioreactor

Since stability was proved stability of the two main interconnections. The chained observer will allow to measure some state or variables of the photobioreactor that are not available in the output. Some difficult variables are measured online in a photobioreactor. In this case, these variables are used to estimate the growth rate and the remanent substrate or nutrients of the culture. In order to achieve this, it is necessary to obtain the linearized model of (16) by using the Jacobian matrix evaluated on the variables and parameters in equilibrium. It is proposed a study a method in order to generate a linearization for any point of equilibrium $x_{est} = [x_{est}\ s_{est}]^T$ and $u(t) = u_{est}$ such as:

$$\dot{x}_l = A_l x_l + b_l u_l \qquad (29)$$
$$y_l = C_l x_l$$

where :

$$A = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_3 \end{pmatrix}\bigg|_{equili}$$

$$b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\bigg|_{equili}$$

$$\alpha_1 = \frac{(a_1 s_{est})}{(a_2 + s_{est})} - u_{est}$$

$$\alpha_2 = \frac{(a_1 s_{est})}{(a_2 + s_{est})} - \frac{(a_1 s_{est} x_{est})}{(a_2 + s_{est})^2}$$

$$\alpha_3 = \frac{(a_1 s_{est})}{a_3(a_2 + s_{est})}$$

$$\alpha_4 = \frac{(a_1 s_{est} x_{est})}{(a_2 + s)^2} - \frac{(a_1 s_{est})}{a_3(a_2 + s_{est})} - u_{est}$$

$$b_1 = -x_{est}$$

$$b_2 = a_4 - s_{est}$$

$$C_l = [1 \quad 0]$$

where the state linear is defined as $x_l = [x - s_{est} \quad s - s_{est}]^T$ and $u_l = u(t) - u_{eq}$. Generally, the stationary

The eigenvalues of $A_l$ are:

$$\lambda_1 = -u_{est} \tag{30}$$

$$\lambda_2 = -\frac{(a_3 s_{est}^2 u_{est} + a_1 a_2 x_{est} - a_1 a_3 s_{est}^2 + a_2^2 a_3 u_{est} + 2a_2 a_3 s_{est} u_{est} - a_1 a_2 a_3 s_{est})}{(a_3 a_2^2 + 2a_3 a_2 s_{est} + a_3 s_{est}^2)}$$

In this work, a wastewater treatment is modeled via a Monod Model. Wastewater samples were obtained from the *Río de los Remedios*, located in the municipality of Ecatepec, State of Mexico, Mexico.

The following values for Monod parameters were used: $a_1 = 0.027 hr^{-1}$, $a_2 = 25 mg/L$, $a_3 = 3.45$, $a_4 = 205.4 mg/L$, $s_0 = 152 mg/L$, $\hat{x}_l = 35 mg/L$, $x_{equili}$. The following microalgae *Spirulina maxima* model is obtained:

$$\frac{dx}{dt} = \mu(s)x - u^{opt}(t)x \tag{31}$$

$$\frac{ds}{dt} = -\frac{1}{3.45}\mu(s)x + u^{opt}(t)(205.4 - s)$$

$$\mu(s) = \frac{0.027s}{25 + s}$$

$$u^{opt}(t) = 0.027(1 - \sqrt{\frac{25}{25 + 205.4}}) = 0.018$$

where $u^{opt}(t)$ is the optimal dilution rate shown in (19).

state is the operation region where a photobioreactor is intended to work for a culture. The stability of this process depends entirely on the dynamics of the optimum dilution rate, which can be checked by calculating the eigenvalue of the linear matrix of (29).

With the above, it is possible to verify the negativity of eigenvalues (30), where the stability of the operating point depends directly on the *u* optimal. In this work $u_{est}$ was calculated using the equation shown in (19). It is proposed that the only measurable variable is biomass, therefore $y_l = C_l x$ with $C = [1 \quad 0]$. In the Monod model shown in (16), the term $\beta(\cdot)$ is unknown. This term cannot be quantifiable and is taken as an uncertainty variable that cannot be modelled on the chained observer.

Some aquatic photosynthetic organisms such as microalgae have environmental applications in water treatment, generally as part of a tertiary wastewater treatment. In Dominguez-Bocanegra A. (2002), the elimination of nutrients (total phosphorus, orthophosphates and ammonia nitrogen) in wastewater is studied by using the microalgae *Spirulina maxima*.

The linearization of the previous system is obtained via stationary states of the variables $x_{est} = 445(mg/L)$, $s_{est} = 4545(mg/L)$ and $u_{est}^{opt}(t) = 0.018(hr^{-1})$.

Calculating the Jacobian matrices via (29) and reducing it is obtained a observer chained for the previous system:

$$\dot{x}_l = Ax_l + bu_v(t) \tag{32}$$

$$\dot{\hat{x}}_0 = A\hat{x}_l + bu_v(t) + Ke_0$$

$$y_l = C_l x_l$$

$$u_v(t) = 0.018 - \hat{\beta}(\cdot)$$

$$\dot{\hat{\beta}}(\cdot) = \kappa_c \dot{E}(t)$$

with $A = \begin{pmatrix} 0 & 0.0624 \\ -0.0052 & -0.0361 \end{pmatrix}$,

$b = \begin{pmatrix} -519.30 & 150.58 \end{pmatrix}^T$ and $C_l = [1 \quad 0]$. Fulfilling conditions indicated in Theorem 1 and calculating in turn the matrix of distribution $\Gamma$. The proportional and integral gains were chosen based on stability (26).

Sucha that $\Gamma$ will be a Hurwitz matrix:

$$\Gamma = \begin{pmatrix} (b_1c_1 + b_2c_2)k_p & (b_1c_1 + b_2c_2) \\ k_i & 0 \end{pmatrix} \quad (33)$$

$$\forall c_1 = 1, c_2 = 0 \rightarrow \Gamma_{C=[1\ 0]} = \begin{pmatrix} -519.30k_p & -519.30 \\ k_i & 0 \end{pmatrix}$$

Such that all eigvalors must be strictly negative for the above Hurwitz matrix, as follows:

$$\lambda_{\Gamma 1} = -259.65k_p - \frac{1}{2}\sqrt{2.6967 \times 10^5 k_p^2 - 2077.2k_i}$$

$$\lambda_{\Gamma 2} = \frac{1}{2}\sqrt{2.6967 \times 10^5 k_p^2 - 2077.2k_i} - 259.65k_p$$

The proportional and integral gains were $k_p$, $k_i = 0.1 \rightarrow \lambda_{\Gamma 1} = -50.2, \lambda_{\Gamma 2} = -1.02$. Therefore, matrix fulfills the condition of *Theorem 1*. Finally, the calculation of matrix $K$ is obtained by calculating values of matrix $A_k = A - K$:

$$K = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$$

$$\rightarrow A_k = \begin{pmatrix} 0 & 0.0624 \\ -0.0052 & -0.0361 \end{pmatrix} - \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$$

$$\lambda_{A_k 1} = -100.01, \lambda_{A_k 2} = -100.03$$

The matrix $A_k$ is Hurwitz with the gains $k_p$, $k_i = 0.1$. The robust chained observer shown in (32) is an observer with practical stability for the model of a photobioreactor (31).

# 3 Results and discussion

## 3.1 *Numerical simulation results*

A numerical study is proposed for the simulation of a *Spirulina maxima* microalgae culture under the parameters shown in the previous section (see Dominguez-Bocanegra A., (2002)). Nutrient estimation (substrate) is verified by comparing the *Chained Observer* shown in (32) with classical non-linear results in (35):

With high gain matrix has the following structure already well studied:

$$S_\infty = \begin{bmatrix} \theta^{-1} & -\theta^{-2} \\ -\theta^{-2} & 2\theta^{-3} \end{bmatrix} \quad (34)$$

$S_\infty \in \mathbb{R}^{2x2}$ is a positive definite symmetric matrix which is the solution of $0 = -hS_\infty - A_o^T S_\infty - S_\infty A_o + C^T C$ through a diffeomorphism and its anti-defeomorphism the next observer of the plant can be obtained (Gauthier *et al.*, 1992):

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{pmatrix} = \begin{pmatrix} (\mu(\hat{s}) - D)\hat{x} - 2\theta\, e(t) \\ -\frac{\mu(\hat{s})}{a_3}\hat{x} + (-\hat{s} + a_4)D + (\frac{2\theta\hat{s}(a_2-\hat{s})}{a_2\hat{x}} - \frac{\theta^2(a_2-\hat{s})^2}{a_1 a_2 \hat{x}})\, e(t) \end{pmatrix} \quad (35)$$

$$\mu(\hat{s}) = \frac{a_1\hat{s}}{a_2 - \hat{s}} \qquad e(t) = \hat{x} - x$$

Based on the parameters suggested by Dominguez-Bocanegra A. (2002), previous system can be expressed as follows:

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{\hat{s}} \end{pmatrix} = \begin{pmatrix} (\mu(\hat{s}) - 0.018)\hat{x} - 2\, e(t) \\ -\frac{\mu(\hat{s})}{3.45}\hat{x} + (-\hat{s} + 205.4)0.018 + (\frac{20\hat{s}(25-\hat{s})}{25\hat{x}} - \frac{100(25-\hat{s})^2}{0.675\hat{x}})\, e(t) \end{pmatrix}$$

$$\mu(\hat{s}) = \frac{0.027\hat{s}}{25 - \hat{s}} \qquad e(t) = \hat{x} - x$$

$$\theta = 10 \quad (36)$$

The high gain observer (HG) shown above has been widely used in the literature. This usually has the problem of requiring a complete knowledge of the system, *i. e.* if nominal parameters are constant or variable. These situation are usually reflected in problems during the estimation. In this work, chained observer (32) was insensitive to a possible change of the parameters $a_1$ and $a_2$ and showed a better performance than classical HG, as it is shown in (35).
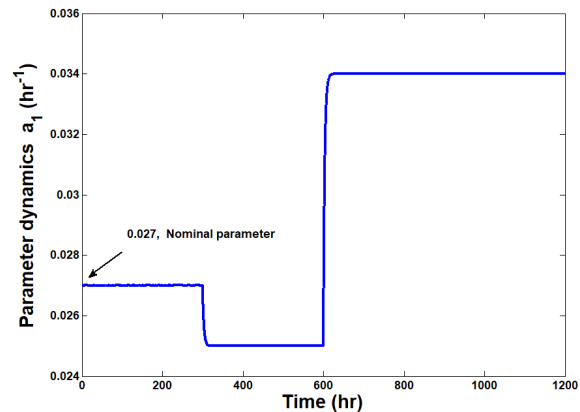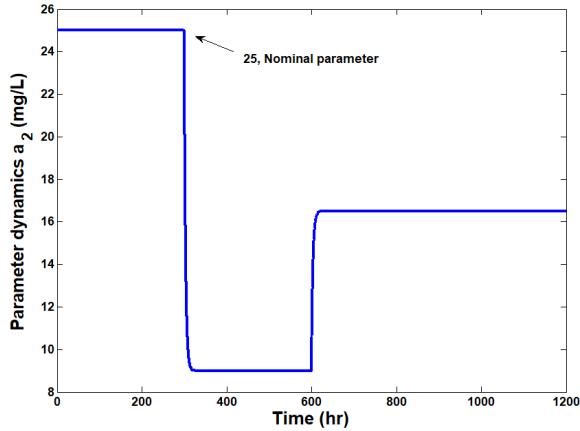
Fig. 2. Parameter dynamics $a_1(hr^{-1})$.

Fig. 3. Parameter dynamics $a_2(mg/L)$.



Fig. 5. Parameter dynamics of sustrate $mg/L$.
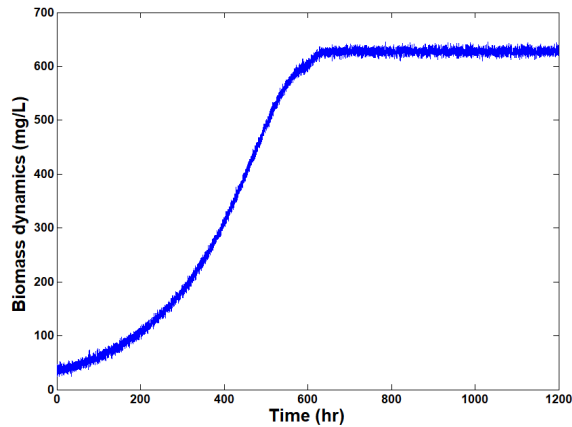


Fig. 6. Parameter dynamics of sustrate $mg/L$.



Fig. 4. Parameter dynamics of biomass.

Therefore, a simulation was carried out in Matlab 7.10 Simulink environment through Dorman-Price numerical method, with a fixed step of 0.001 and 1200 hours of culture process time of *Spirulina maxima*. A change in Monod's parameters ($a_1, a_2$) is proposed (see to figure 2 y 4). In addition, an output signal with Gaussian noise (sensor dynamics) is proposed in such a way that it recreates real conditions (Figure 4).

Numerical simulation is shown in figure 5. These results demonstrated that the chained observer (25) was superior to a non-linear HG observer because the chained observer is insensitive to the changes in the operating parameters shown in (figure 5) even though the technique is relatively much less complex than the HG from a point of view of automatic control theory.
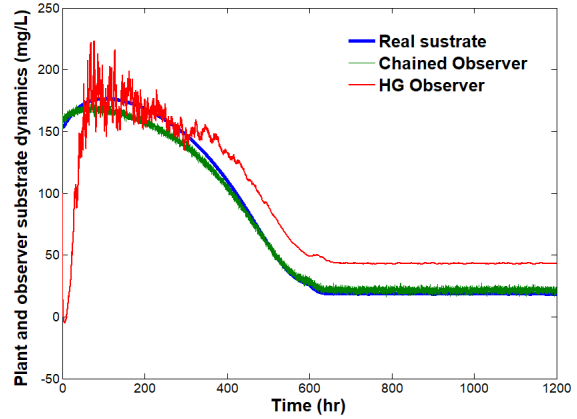


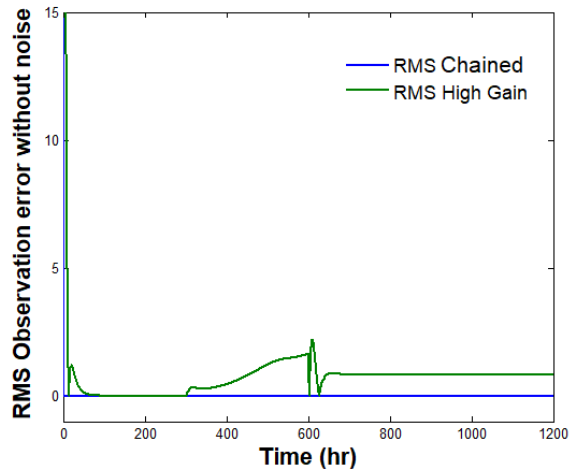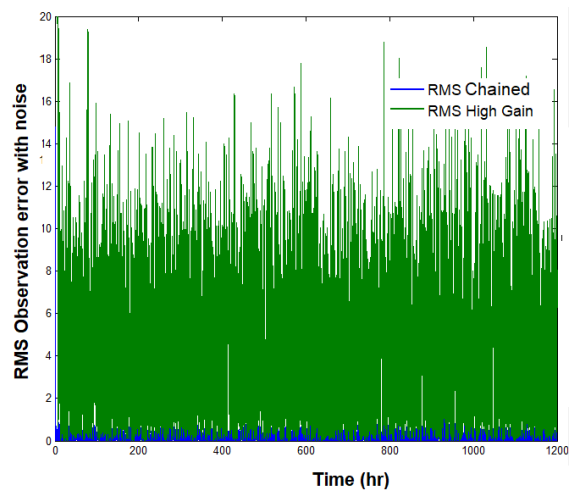Fig. 7. Parameter dynamics of sustrate $mg/L$.

For a better analysis of the effectiveness of the proposed technique, a root mean square (RMS) analysis is proposed. With the aim of a more crucial analysis, an analysis with noise and another without noise is shown, since as we know, the noise in the signal increases significantly the REM, this with the aim of a better comparison.

It is easy to see that based on these results in the RMS calculation, that the chained observer is superior without noise to the high gain observer, it is also possible to see the most known disadvantages of the high gain value, in the appreciation of the noise at the output the RMS grows critically while in the chained observer remains within much lower ranges (see figures 7 and 8).

## Conclusions

This paper deals with the study of a photobioreactor modeling, with some environmental disturbances and the common problems present while using analytical tools for photobioreactor control. It was demonstrated that observers can have a great application to multidisciplinary tasks such as estimating variables that can not be measured on-line through common methods, such as nutrients in a microalgae culture. A special type of observer was proposed for biological systems, the so-called Chained observer. Under certain conditions, this observer presented better results than the HG nonlinear control. Dynamic nutrient estimation was tested in a wastewater treatment model using *Spirulina maxima* culture. The chained observer supported the control of microalgae cultivation process, which can be extended to other cultures and/or types of photobioreactors.

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