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ON THE EXPLICIT EXPRESSIONS FOR THE DETERMINATION OF THE FRICTION FACTOR IN TURBULENT REGIME

ACERCA DE LAS EXPRESIONES EXPLÍCITAS PARA LA DETERMINACIÓN DEL COEFICIENTE DE FRICCIÓN EN RÉGIMEN TURBULENTO

J.R. Pérez Pupo[1*], M.N. Navarro-Ojeda[2], J.N. Pérez-Guerrero[3], M.A. Batista-Zaldívar[4]

[1]*Polytechnic Higher School of Chimborazo (ESPOCH). Industrial Engineering School. Panamerican South Avenue. Km 1$\frac{1}{2}$, Riobamba, Ecuador.*
[2]*Polytechnic Higher School of Chimborazo (ESPOCH). Faculty of Animal Sciences. Panamerican South Avenue. Km 1$\frac{1}{2}$, Riobamba, Ecuador.*
[3]*Manabí Technical University (UTM). Faculty of Mathematical, Physical and Chemical Sciences. Urbina and Che Guevara Avenue, Porto Viejo, Manabí, Ecuador.*
[4]*Manabí Technical University (UTM). Chemistry Department. Institute of Basic Sciences. Urbina and Che Guevara Avenue, Porto Viejo, Manabí, Ecuador.*

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Abstract
Pressure failure in pipelines is commonly calculated by the Darcy-Weisbach formula. To use this formula, the Darcy's hydraulic friction factor must be known. The best approximation for the Darcy's friction coefficient in turbulent flow regime, is expressed by the Colebrook-White equation. This equation can only be solved by numerical methods, because it is implicit for the friction coefficient. There are other approximate explicit models for the calculation with different relative errors in comparison to the Colebrook-White equation. In this work, a review of the friction factor explicit correlations is made including, for the first time in an article of this type, 48 equations, 10 of them that had not been reported previously and other explicit equations whose precision does not agree with previous reports or had not been included in review articles on this topic. The precision of these equations was determined by means of the maximum relative error for Reynolds numbers and relative roughness, considered as the equation's application range. As a result of this work, the evaluation and selection criteria to use the explicit expressions for the calculation of the friction coefficient in turbulent regime is offered to engineering professionals and students.
Keywords: Hydraulic friction coefficient, turbulent regime, explicit models, relative roughness, explicit correlations.

Resumen
La caída de presión en las tuberías se calcula mediante la fórmula Darcy-Weisbach. Para utilizar esta fórmula, debe conocerse el coeficiente de fricción hidráulica de Darcy. La mejor aproximación al coeficiente de fricción de Darcy, para el flujo en régimen turbulento, viene dada por la ecuación de Colebrook-White. Esta ecuación solo puede resolverse mediante métodos numéricos, pues es implícita para el coeficiente de fricción. Hay varios otros modelos explícitos aproximados para su determinación con diferentes errores relativos en comparación con la ecuación de Colebrook-White. En este estudio, se realizó una revisión de las correlaciones explícitas del factor de fricción, y se incluyen, por primera vez en un artículo de este tipo, 48 ecuaciones, de ellas, 10 que no habían sido reseñadas con anterioridad y otras ecuaciones explicitas con cuya precisión no se concuerda con reportes previos, o no se habían incluido en artículos de revisión sobre la temática; se determinó su precisión a través del error relativo máximo, para los números de Reynolds y rugosidades relativas consideradas como su rango de aplicación. Como resultado del trabajo, se brindan a los profesionales y estudiantes de ingeniería, los criterios de evaluación y selección de las expresiones explícitas para el cálculo del coeficiente de fricción en flujo turbulento.
Palabras clave: Coeficiente de fricción hidráulica, régimen turbulento, modelos explícitos, rugosidades relativas, correlaciones explícitas.

* *Corresponding author. E-mail*: `juan.perezp@espoch.edu.ec`
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1 Introduction

The determination of the hydraulic friction factor λ value in the general case, depends on the liquid flow regime and on the pipeline material properties. When the flow regime is laminar or turbulent and the pipeline is smooth, the expressions for its calculation coincide and there are no major difficulties; but, when the regime is turbulent, as it is commonly, and the pipeline is rough, there is a great diversity of expressions for the calculation of this coefficient.

The determination of the hydraulic friction factor value λ for the general case, depends on the liquid's regime flow and on the pipeline's material properties. For the calculation of this coefficient in fluid transport systems, which generally operate in turbulent flow regime, there are many formulas proposed.

To calculate λ coefficient in turbulent regime, different authors have proposed a great number of calculation formulas. Initially it was assumed that this coefficient was constant (for example, according to to Dupuit, λ= 0.0025); afterwards various formulas were obtained based in the experimental data processing (Pérez, 2002).

Since 1945, engineers, academics and engineering students have been using a friction factor diagram for the pipeline flow as the type published by Moody, 1947. This diagram is semi-empirical and the conditions that it describes (completely developed flow, isothermal, uncompressible, dissipative, quasi stationary), are never enough reached in practical real cases (McGovern, 2011). Although there has been considerable progress in the calculation of surface roughness, especially with the aid of computational methods, and in the measurement and comprehension of velocities' distribution in the limit layers, the diagram is still considered as a temporary solution and has the function that Moody stated: "...a simple way to estimate friction factors" (McGovern, 2011).

From the bibliographic analysis made in this work, it is observed that there are two types of approaches for the calculation: the implicit calculations, where the Colebrook-White formula is predominant, and the explicit calculations, where there is a great diversity of expressions and disagreements with respect to their complexity and precision, between the researchers that have addressed the theme.

The Colebrook-White formula (Colebrook,1939), one of the first and more frequently used expressions to calculate the λ coefficient in turbulent regime, is given by equation (1), where the ε/D relation represents the pipeline's relative roughness.

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{\frac{\varepsilon}{D}}{3,7} + \frac{2,51}{Re\sqrt{\lambda}}\right) \tag{1}$$

The use of this formula is not convenient because the model is implicitly defined in the friction factor and iterations are needed for the calculation. To find the friction factor implicitly stated in the Colebrook-White equation, the use of numerical algorithms is needed, which is not as rapid as approximations. Particularly in complex and supercritical pipelines flow systems, its use becomes difficult, and that is the reason why its use is not recommended in practical engineer's calculations or by students in courses related to fluids mechanics.

Difficulties to determine λ by means of Colebrook-White equation, have forced many researchers over the world (Rohsenow, Hartnett, & Cho, 1998), to make efforts in developing explicit equations that could be used alternatively: ones simpler and compact, easier to memorize but with great deviations; others less compact and complex, more difficult to memorize but with small deviations and, some others which combine simplicity and precision, with very reduced errors in the friction factor compared with the calculated value using the Colebrook-White equation (Diniz and Souza, 2009). The explicit expressions for the calculation of the friction coefficient in turbulent flow, most frequently used and cited in literature, are those of Halland and Swamee-Jain (Eq.12 and 20); meanwhile there is an important and interesting group of these equations (Eq. 2, 4, 5, 6, 7, 26, 29, 46, 47 and 48), that are not analyzed in the articles here reviewed, being many of these proposals evaluated as alternatives to the traditionally used ones.

In summary, it was observed that there is no agreement with respect to the expressions' complexity and precision, there is lack of analysis of some expressions cited in literature and also, there is absence of calculation criteria and analysis of the advantages and disadvantages of their use. This lack of information leads to some degree of confusion between technicians and students when they need to choose the expression to use.

As a result of this study, a critical exhaustive review of 48 explicit correlations for the calculation of the hydraulic friction coefficient in turbulent flow is presented, with the objective of analyzing their complexity and precision, including some less known explicit correlations, to provide engineer professionals and engineering students, a set of evaluation and selection criteria for practical uses.

2 Materials and methods

An exhaustive and critical review was made of 68 research works on the subject of the calculation of the turbulent flow friction factor, published in English, Spanish, German, Russian and Portuguese, in recognized indexed journals from data bases in the field of study. The search strategy used as criterion of inclusion, was that consulted sources had to verse on explicit expressions for the calculation or the valuation of their precision, which led to the identification and analysis of 48 friction factor's correlations that cover a wide range of relative roughness and Reynolds numbers. Works related with mathematical methods for the solution of implicit equations for the friction factor calculation and the equations that are valid only for smooth pipelines or non-Newtonian fluids, were excluded. From the use of documentary analysis tools for primary information, it was assumed as a rule of the logic of the results reported in this work, to present the chronological order of publications with conclusions and analytical-synthetic deductions.

3 Results and discussion

Published articles in indexed journals are the main communication medium for the results of research on the friction factor calculation in turbulent flow, as they represent the 88.1% of the total consulted sources (Table 1). Most of the articles have been published in journals in English language, and they represent the 77.6% of the total of consulted articles. These articles appear in many journals, but 8 of these journals publish the 37.8% of the total number of articles (Table 2).

Table 1. Bibliographic sources consulted.

Consulted sources	Quantity
Journal articles	56
Books	8
Reports	2
Conferences	1
Book chapters	1
Non published work	1

Table 2. Journals with major number of publications in the topic.

Journals	Number of articles
Journal of Hydraulic Engineering	3
International Journal of Heat and Mass Transfer	2
Journal of Petroleum Science and Engineering	2
AIChE Journal	2
Advances in Engineering Software	2
Journal of Fluids Engineering	2
Journal of the Hydraulics Division	2
Industrial & Engineering Chemistry Fundamentals	2

Table 3. Authors with major number of publications.

Authors	Number of articles
Brkić, D.	4
Churchill, S. W.	2
Jain, A. K.	2
Zigrang, D. J.	2
Sylvester, N. D.	2
Offor, U. H.	2
Alabi, S. B.	2

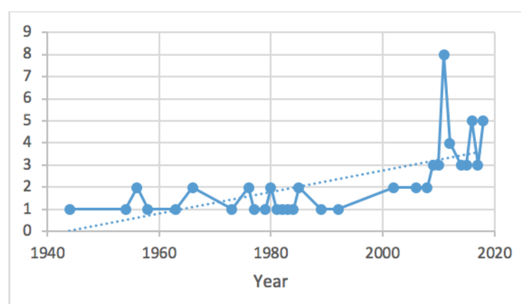

Fig. 1. Distribution of publications per year.

There are many researchers that have published works on the calculation of friction factor in turbulent flow, but there are 7 authors who publish more frequently and have written two or more articles than other researchers on this topic (Table 3). In the last nine decades, from 1940 to our days, there has been an increasing research interest in this topic, especially in the last ten years as, until 2008 there was a uniform, nearly constant, not increasing tendency (Fig. 1).

Table 4. Explicit expressions for the calculation of the Friction Coefficient in Turbulent Flow.

Equation number	**Author (Reference)**	**Year**	**Equation**	**Application range (or valuation)**
1	Moody (1947)	1947	$\lambda = 0.0055\left[1+\left(2x10^{4}\frac{\varepsilon}{D}+\frac{10^{6}}{Re}\right)^{1/3}\right]$	$4\times10^{3} \leq Re \leq 10^{8}$ $0 \leq \varepsilon/D \leq 10^{-2}$
2	Altshul (1963)	1950	$\frac{1}{\sqrt{\lambda}} = 1.8log\left(\frac{Re}{0.1Re\frac{\varepsilon}{D}+7}\right)$	$4\times10^{3} \leq Re \leq 1x10^{6}$ $1.6x10^{-4} \leq \varepsilon/D \leq 2.5x10^{-2}$
3	Altshul (1963)	1950	$\lambda = 0.11\left(\frac{68}{Re}+\frac{\varepsilon}{D}\right)^{0,25}$	$4\times10^{3} \leq Re \leq 1x\ 10^{6}$ $1.6x10^{-4} \leq \varepsilon/D \leq 2.5x10^{-2}$
4	Agroskin (1954)	1951	$\frac{1}{\sqrt{\lambda}} = -1.8.log\left[\frac{6.8}{Re}+\left(\frac{\varepsilon}{3.7D}\right)^{1,1}\right].$	$4\times10^{3} \leq Re \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 10^{-2}$
5	Frenkel (1956)	1951	$\frac{1}{\sqrt{\lambda}} = -2.log\left(\left(\frac{6.81}{Re}\right)^{0,9}+\frac{\varepsilon}{3.7D}\right)$	$4\times10^{3} \leq Re \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 10^{-2}$
6	Lobaev (1956)	1956	$\lambda = \frac{1.42}{\left[log\left(Re\frac{D}{\varepsilon}\right)^{2}\right]}$	$1x10^{-4} \leq \varepsilon/D \leq 1\times10^{-2}$
7	Chernikin (1958)	1958	$\frac{1}{\sqrt{\lambda}} = -1.83.\,log\left[\frac{8.5}{Re}+\left(\frac{\varepsilon}{3.7D}\right)^{1.093}\right]$	$4\times10^{3} \leq Re \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 10^{-2}$
8	Wood (1966)	1966	$\lambda = 0.094\left(\frac{\varepsilon}{D}\right)^{0.225}+0.53\left(\frac{\varepsilon}{D}\right)+88\left(\frac{\varepsilon}{D}\right)^{0.44}Re^{-\left[1.62\left(\frac{\varepsilon}{D}\right)^{0.134}\right]}$	$4\times10^{3} \leq Re \leq 5\times10^{7}$ $10^{-5} \leq \varepsilon/D \leq 4\times10^{-2}$
9	Churchill (1973)	1973	$\frac{1}{\sqrt{\lambda}} = -2log\left[\frac{\varepsilon}{3.7D}+\left(\frac{7}{Re}\right)^{0.9}\right]$	Not specified
10	Eck (1966)	1963	$\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{\varepsilon}{3.715D}+\frac{15}{Re}\right)$	$4\times10^{3} \leq Re \leq 10^{8}$ $5x10^{-6} \leq \varepsilon/D \leq 10^{-2}$
11	Jain (1976)	1976	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.715D}+\left(\frac{6.943}{Re}\right)^{0.9}\right]$	$5\times10^{3} \leq Re \leq 10^{7}$ $4x10^{-5} \leq \varepsilon/D \leq 5x10^{-2}$
12	Swamee and Jain (1976)	1976	$\lambda = \frac{0.25}{\left[log\left(\frac{\varepsilon/D}{3.7}+\frac{5.74}{Re^{0.9}}\right)\right]^{2}}$	$5\times10^{3} \leq Re \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$
13	Churchill (1977)	1977	$\lambda = 8\left[\left(\frac{8}{Re}\right)^{12}+\frac{1}{(A+B)^{1.5}}\right]^{\frac{1}{12}}$ $A = \left[2.457\ln\left(\frac{1}{\left(\frac{7}{Re}\right)^{0.9}+\left(\frac{0.27\varepsilon}{D}\right)}\right)\right]^{16}$ $B = \left(\frac{37530}{Re}\right)^{16}$	For all flow regimes and relative roughness

Equation number	**Author (Reference)**	**Year**	**Equation**	**Application range (or valuation)**
14	Chen (1979)	1979	$\frac{1}{\sqrt{\lambda}}=-2log\left[\frac{\varepsilon}{3.7065D}-\frac{5.0452}{Re}log\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098}+\frac{5.8506}{Re^{0.8981}}\right)\right.$	$4\times10^3 \leq Re \leq 4\times10^8$ $10^{-7} \leq \varepsilon/D \leq 5\times10^{-2}$
15	Round (1980)	1980	$\frac{1}{\sqrt{\lambda}}=1.8log\left[\frac{Re}{0.135Re\left(\frac{\varepsilon}{D}\right)+6.5}\right]$	$4\times10^3 \leq Re \leq 10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
16	Schorle and col. (1980)	1980	$\frac{1}{\sqrt{\lambda}}=-2log\left[\frac{\varepsilon}{3.7D}-\frac{5.02}{Re}log\left(\frac{\varepsilon}{3.7D}+\frac{14.5}{Re}\right)\right]$	$4\times10^3 \leq Re \leq 4\times10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
17	Barr and White (1981)	1981	$\frac{1}{\sqrt{\lambda}}=-2log\left\{\frac{\varepsilon}{3.7D}+\frac{4.518log\left(\frac{Re}{7}\right)}{Re\left[1+\frac{Re^{0.52}}{29}\left(\frac{\varepsilon}{D}\right)^{0.7}\right]}\right\}$	Not specified
18	Zigrang and Sylvester (1982)	1982	$\frac{1}{\sqrt{\lambda}}=-2log\left\{\frac{\varepsilon}{3,7D}-\frac{5,02}{\mathrm{Re}}\log\left[\frac{\varepsilon}{3,7D}-\frac{5,02}{\mathrm{Re}}\log\left(\frac{\varepsilon}{3,7D}+\frac{13}{\mathrm{Re}}\right)\right]\right\}$	$4\times10^3 \leq Re \leq 10^8$ $4\times10^{-5} \leq \varepsilon/D \leq 5\times10^{-2}$
19	Zigrang and Sylvester (1982)	1982	$\frac{1}{\sqrt{\lambda}}=-2,0log\left\{\frac{\varepsilon/D}{3,7}-\frac{5,02}{Re}log\left[\frac{\varepsilon/D}{3,7}-\frac{5,02}{Re}log\left[\frac{\varepsilon/D}{3,7}-\frac{5,02}{Re}\right]\log\left(\frac{\varepsilon/D}{3,7}+\frac{13}{Re}\right)\right]\right.$	$4\times10^3 \leq Re \leq 10^8$ $4\times10^{-5} \leq \varepsilon/D \leq 5\times10^{-2}$
20	Haaland (1983)	1983	$\lambda=-1.8log\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right)^{-2}$	$4\times10^3 \leq Re \leq 10^8$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$
21	Serghides (1984)	1984	$\frac{1}{\sqrt{\lambda}}=A-\frac{(B-A)^2}{C-2B+A}$ $A=-2log\left[\left(\frac{(\varepsilon/D)}{3.7}\right)+\frac{12}{Re}\right]$ $B=-2log\left[\left(\frac{\varepsilon/D}{3.7}\right)+\frac{2.51A}{Re}\right]$ $C=-2log\left[\left(\frac{(\varepsilon/D)}{3.7}\right)+\frac{2.51B}{Re}\right]$	For all the flow regimes and relative roughness
22	Serghides (1984)	1984	$\lambda=\left[4.781-\frac{(A_7-4.781)^2}{A_8-2A_7+4.781}\right]^{-2}$ $A_7=-2\log\left(\frac{\varepsilon}{3.7D}+\frac{12}{\mathrm{Re}}\right)$ $A_8=-2\log\left(\frac{\varepsilon}{3.7D}+\frac{2.51A_7}{\mathrm{Re}}\right)$	Not specified

Equation number	**Author (Reference)**	**Year**	**Equation**	**Application range (or valuation)**
23	Tsal (1989)	1989	$A = 0.11\left(\frac{68}{Re}+\frac{\varepsilon}{D}\right)^{0.25}$ $If\ \ A \geq 0.018\ then\, \lambda = A$ $If\ \ A < 0.018\ then\, \lambda = 0.0028 + 0.85A$	$4\times10^{3} \leq \mathrm{Re} \leq 10^{8}$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
24	Robaina (1992)	1992	$\frac{1}{\sqrt{\lambda}} = -2log\left(0.27\frac{\varepsilon}{D}+\frac{5.62}{Re^{0.9}}\right)$	$4\times10^{3} \leq \mathrm{Re} \leq 4x10^{7}$ $10^{-5} \leq \varepsilon/D \leq 10^{-2}$
25	Manadilli (1997)	1997	$\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{\varepsilon}{3.7D}+\frac{95}{Re^{0.983}}+\frac{96.82}{Re}\right)$	$5{,}235\times10^{3} \leq \mathrm{Re} \leq 10^{8}$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
26	Chernikin and Chernikin (2012)	1997	$\lambda = 0.11\left[\frac{\alpha+\delta+X^{1,4}}{115X+1}\right]^{0.25}$ $\alpha = \frac{68}{Re};\ \ \delta = \frac{k}{D};\ \ \ X = (28\alpha)^{10}$	For all flow regimes $1.6x10^{-4} \leq \varepsilon/D \leq 2.5x10^{-2}$
27	Sousa and col. (1999)	1999	$\frac{1}{\sqrt{\lambda}} = -2log\left[\frac{\varepsilon}{3.7D}-\frac{5.16}{Re}log\left(\frac{\varepsilon}{3.7D}+\frac{5.09}{Re^{0.87}}\right)\right]$	$4\times10^{3} \leq \mathrm{Re} \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 10^{-2}$
28	Romeo and col. (2002)	2002	$\frac{1}{\sqrt{\lambda}} = -2log\left\{\frac{\varepsilon}{3.7065D}-\frac{5.0272}{Re}log\left[\frac{\varepsilon}{3.827D}-\frac{4.567}{Re}log(A)\right]\right.$ $A = \left(\frac{\varepsilon}{7.7918D}\right)^{0.9924} + \left(\frac{5.3326}{208.815\,\mathrm{Re}}\right)^{0.9345}$	$3\times10^{3} \leq \mathrm{Re} \leq 1.5\times10^{8}$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
29	Dobromyslov (2004)	2004	$\sqrt{\lambda} = 0.5\frac{\frac{b}{2}+\frac{\left(1.312(2-b)log\left(\frac{3.7D}{\varepsilon}\right)\right)}{lg(Re)-1}}{log\left(\frac{3.7D}{\varepsilon}\right)}$ $b = 1+\frac{log(Re)}{log(Re_{kv})}$ $If\ \ b>2,\ \ b=2 \qquad Re_{kv} = 500.\frac{D}{\varepsilon}$	Not specified
30	Sonnad and Goudar (2006)	2006	$\frac{1}{\sqrt{\lambda}} = 0.8686ln\left(\frac{0.4587Re}{G^{(G/G+1)}}\right)$ $\mathrm{G} = 0.1240\times\frac{\varepsilon}{\mathrm{D}}\times\mathrm{Re}+\ln(0.4587\mathrm{Re})$	$4\times10^{3} \leq \mathrm{Re} \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$
31	Rao and Kumar (2007)	2006	$\frac{1}{\sqrt{\lambda}} = 2log\left[\frac{\left(2\frac{\varepsilon}{D}\right)^{-1}}{\left(\frac{0.444+0.135Re}{Re}\right)\beta}\right]$ $\beta = 1-0.55e^{-0.33\left[In\left(\frac{Re}{6.5}\right)\right]^{2}}$	$2300 \leq \mathrm{Re} \leq 10^{8}$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$

Equation number	Author (Reference)	Year	Equation	Application range (or valuation)
32	Buzzelli (2008)	2008	$\frac{1}{\sqrt{\lambda}} = B_1 - \left[\frac{B_1 + 2log\left(\frac{B_2}{Re}\right)}{1+\frac{2.18}{B_2}}\right]$ $B_1 = \frac{[0.774ln(Re)]-1.41}{\left(1+1.32\sqrt{\frac{\varepsilon}{D}}\right)}$ $B_2 = \frac{\varepsilon}{3.7D}Re + 2.51B_1$	$3\times10^3 \leq Re \leq 1.5\times10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
33	Vantankhah and Kouchakzadeh (2008)	2008	$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\left[\frac{0.4587Re}{(S-0,31)^{\left(\frac{S}{S+0.9633}\right)}}\right]$ $S = 0.124Re\left(\frac{\varepsilon}{D}\right) + ln(0.4587Re)$	$4\times10^3 \leq Re \leq 10^8$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$
34	Avci and Karagoz (2009)	2009	$\lambda = \frac{6.4}{\left\{ln(Re) - ln\left[1+0.01Re\frac{\varepsilon}{D}\left(1+10\sqrt{\frac{\varepsilon}{D}}\right)\right]\right\}^{2.4}}$	$2300 \leq Re \leq 10^8$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$
35	Papaevangelou and col. (2010)	2010	$\lambda = \frac{0.2479 - 0.0000947(7-logRe)^4}{\left[log\left(\frac{\varepsilon}{3.615D} + \frac{7.366}{Re^{0.9142}}\right)\right]^2}$	$10^4 \leq Re \leq 10^7$ $10^{-5} \leq \varepsilon/D \leq 10^{-2}$
36	Chernikin and Talipov (2010)	2010	$\lambda = 0{,}168\frac{(\frac{\varepsilon}{D})^{0.102}}{Re^{0.113}}$	$8x10^{-5} \leq \varepsilon/D \leq 1x10^{-3}$ $10^4 \leq Re \leq 4x10^6$
37	Brkić (2011a)	2011	$\frac{1}{\sqrt{\lambda}} = -2log\left(10^{-0.4343\beta} + \frac{\varepsilon}{3.71D}\right)$ $\beta = ln\frac{Re}{1.816ln\left[\frac{1.1Re}{ln(1+1.1Re)}\right]}$	$4\times10^3 \leq Re \leq 10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
38	Brkić (2011a)	2011	$\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{2.18\beta}{Re} + \frac{\varepsilon}{3.71D}\right)$ $\beta = ln\frac{Re}{1.816ln\left[\frac{1.1Re}{ln(1+1.1Re)}\right]}$	$4\times10^3 \leq Re \leq 10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
39	Fang and col. (2011)	2011	$\lambda = 1.613\left\{ln\left[0.234\left(\frac{\varepsilon}{D}\right)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right]\right\}^{-2}$	$3\times10^3 \leq Re \leq 10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
40	Ghanbari and col. (2011)	2011	$\lambda = \left\{-1.52log\left[\left(\frac{\varepsilon}{7.21D}\right)^{1.042} + \left(\frac{2.731}{Re}\right)^{0.9152}\right]\right\}^{-2.169}$	$2.1\times10^3 \leq Re \leq 10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$

Equation number	Author (Reference)	Year	Equation	Application range (or valuation)
41	Samadianfard (2012)	2012	$\lambda=\left(\frac{Re^{\frac{\varepsilon}{D}}-0.6315093}{Re^{\frac{1}{3}}+\mathrm{Re}.\frac{\varepsilon}{D}}\right)+0.02275308\left(\frac{6.929841}{Re}+\frac{\varepsilon}{D}\right)^{\frac{1}{9}}+A$ $A=\left(\frac{10^{\frac{\varepsilon}{D}}}{\frac{\varepsilon}{D}+4.781616}\right)\left(\sqrt{\frac{\varepsilon}{D}}+\frac{9.99701}{Re}\right)$	$4x10^3 \leq \mathrm{Re} \leq 10^8$ $10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$
42	Shaikh and col. (2015)	2015	$\lambda=0.25\left[log\left(\frac{2.51}{\alpha Re}+\frac{\varepsilon}{3.7D}\right)\right]^{-2}$ $\alpha=\left[1.14-2log\left(\frac{\varepsilon}{D}\right)\right]^{-2}$	$10^4 \leq \mathrm{Re} \leq 10^8$ $10^{-4} \leq \varepsilon/D \leq 5\times10^{-2}$
43	Brkić (2016)	2016	$\frac{1}{\sqrt{\lambda}}=-2log\left[\frac{2.51\left(1.14-2log\left(\frac{\varepsilon}{D}\right)\right)}{Re}+\frac{\varepsilon}{3.71D}\right]$	$10^4 < \mathrm{Re} < 10^8$ $10^{-6} < \varepsilon/D < 5\times10^{-2}$
44	Offor and Alabi (2016a)	2016	$\frac{1}{\sqrt{\lambda}}=-2log\left\{\frac{\varepsilon}{3.71D}-\frac{1.975}{Re}\left[ln\left(\left(\frac{\varepsilon}{3.93D}\right)^{1.092}+\left(\frac{7.627}{Re+395.9}\right)\right)\right]\right\}$	$4\times10^3 \leq \mathrm{Re} \leq 10^8$ $0 \leq \varepsilon/D \leq 5\times10^{-2}$
45	Beluco and Schettini (2016)	2016	$\lambda=\frac{0.3009}{\left[log\left(\left(\frac{\varepsilon}{3.7315D}\right)^{1.0954}+\left(\frac{5.9802}{Re}\right)^{0.9695}\right)\right]^2}$	$3x10^3 < \mathrm{Re} < 9x10^8$ $0 < \varepsilon/D < 9\times10^{-2}$
46	Brkić and Praks (2018)	2018	$\lambda=\overset{(a)}{\frac{64}{Re}.(1-y_1)}+\overset{(c_1)}{\frac{0.316}{Re^{0.25}}.(y_1-y_3)}+\overset{(c_3)}{\frac{0.25}{log^2\left(\frac{\varepsilon}{3.71}\right)}.y_2}$ I $\lambda=\underset{(a)}{\frac{64}{Re}.(1-y_1)}+\underset{(c_1)}{\left(0.0032+\frac{0.221}{Re^{0.237}}\right).(y_1-y_3)}+\underset{(c_3)}{0.11.\varepsilon^{0.25}.y_2}$ II	For all flow regimes and relative roughness
47	Brkić and Praks (2018)	2018	$\lambda=\overset{(a)}{\frac{64}{Re}.(1-y_1)}+\overset{(c_1)}{\frac{0.316}{Re^{0.25}}.(y_1-y_3)}+\overset{(c_3)}{0.11.\varepsilon^{0.25}.y_2}$ III $\lambda=\underset{(a)}{\frac{64}{Re}.(1-y_1)}+\underset{(c_1)}{\left(0.0032+\frac{0.221}{Re^{0.237}}\right).(y_1-y_3)}+\underset{(c_3)}{\frac{0.25}{log_{10}{}^2\left(\frac{\varepsilon}{3.71}\right)}.y_2}$ IV $y_1=1-\frac{1048}{\frac{4.489}{10^{20}}.Re^6.\left(0.148.Re-\frac{2.306.Re}{0.003133.Re+9.646}\right)+1050}$ $y_2=1.012-\frac{1}{0.02521.Re.\varepsilon+2.202}$ $y_3=1-\frac{1}{0.000389.Re^2.\varepsilon^2+0.0000239.Re+1.61}$	

Equation number	Author (Reference)	Year	Equation	Application range (or valuation)
48	Azizi and col. (2018)	2018	$\lambda=\left[-2ln\left(\frac{\frac{\varepsilon}{D}}{3.7}+\frac{8.147}{Re^{0.928}}\right)\right]^{-2.096}.\frac{1+A+B}{1+C+D}$ $A=a.\left[ln(Re)\right]^{0.01}+b.\left[ln(Re)\right]^{0.1}$ $+c.ln(Re)+d.ln\left(\frac{\varepsilon}{D}\right)+e.\left[ln\left(\frac{\varepsilon}{D}\right)\right]^{2}+f.\left[ln\left(\frac{\varepsilon}{D}\right)\right]^{3}+g.ln(Re)ln\left(\frac{\varepsilon}{D}\right)$ $B=h.(Re)^{-0.4}+i.\left(\frac{\varepsilon}{D}\right)^{0.5}+j.(Re)^{0.3}\left(\frac{\varepsilon}{D}\right)^{1.4}$ $C=k.ln(Re)+l.\left[ln(Re)\right]^{0.3}+m.ln\left(\frac{\varepsilon}{D}\right)+n.\left[ln\left(\frac{\varepsilon}{D}\right)\right]^{2}+o.\left[ln(Re)\right]^{2.3}ln\left(\frac{\varepsilon}{D}\right)$ $D=p.\left(\frac{\varepsilon}{D}\right)^{0.695}+q.\left(\frac{\varepsilon}{D}\right)^{1.67}+r.\left(\frac{\varepsilon}{D}\right)^{2.6}$ a= 27,6174133619768 b= -25,6353346801891 c= 0,349879778909867 d= -6,0745613327875E-03 e= 4,47951990420369 E-04 f= -6,38574665338776E-05 g= -4,6057036243026E-03 h= -3,18061755452957 i= 0,329282387073743 j= -2,62873832656825E-03 k= 0,057457596107698 l= -0,746573202915635 m= -2,18201121395341E-03 n= 4,3570542661899E-04 o= -3,45044088227631E-06 p= 4,43511785774609E-02 q= -1,97409129131745 E-02 r= 0,179513508976549	$10^{-6}\leq\frac{\varepsilon}{D}\leq 0.05$ $2x10^{3}\leq Re\leq 10^{8}$
49	Azizi and col. (2018)	2018	$\lambda=\left[1.805log\left(\frac{\left(\frac{\varepsilon}{D}\right)^{1.108}}{4.267}+\frac{5.164}{Re^{0.966}}\right)\right]^{-2}$	$4x10^{3}\leq Re\leq 10^{8}$ $10^{-6}\leq \varepsilon/D\leq 5\times10^{-2}$

Many explicit approximations of the loss of charge coefficient with different precision and complexity levels, have been developed -and keep developing-, for the substitution of the Colebrook-White standard implicit equation.

Table 4 shows in chronological order, the 48 explicit expressions found in this study with their application range, including 10 formulas that had not been previously described in none of the review articles consulted. Recent expressions as the proposals for Zeghadnia, Robert & Achour (2019) have not been incorporated, because they consist fundamentally in corrections of some of those already referred to in table 4.

Explicit approximations give a relatively good prediction of the friction factor λ and they can reproduce with precision the Colebrook-White equation and Moody's diagram. Generally, models with more complex approximations are the most accurate (Brkić y Ćojbašić, 2017).

The existing explicit models precisions have been evaluated by three statistical criteria: the mean square error, the percent relative error and the absolute error (Zigrang and Sylvester, 1985; Yildirim, 2009; Jaric *et al*., 2011; Winning and Coole, 2013), and additionally, by the model of selection criteria (MSC) and the Akaike's information criterion (AIC). The two last criteria were used by Romeo, Royo and Monzon (Romeo *et al*., 2002) for the selection of the model.

Table 5. Main research work on the topic in the last ten years.

Author (s)	Description	Results
Yildirim (2009)	An extense comparison test was stablished for a wide range of relative roughness (ε/D) and Reynolds (Re) ($1.10^{-6} \leq \varepsilon/D \leq 5.10^{-2}$; $4.103 \leq \text{Re} \leq 1.10^{8}$), that covers a great part of turbulent flow zone in the Moody Diagram.	The majority of the approximations provide estimations of the friction factor with an absolute mean error of 5x10-4, absolute maximum error of $4x10^{-3}$, mean relative error of 1.3 % and a maximum relative error of 5.8 %; in the complete range of values of ε/D and Re. The comparative analysis for the mean relative error profile, the classification of the six best adjusted equations revised, were in good agreement with those of the best model selection criterion, vindicated in recent literature, for the simulations made.
Jaric *et al*. (2011)	Review of the most common explicit correlations for the estimation of the friction factor in smooth a rough pipeline. The comparison of friction factor equations with Colebrook-White equation, was expressed by means of the mean relative error, the positive maximum error, the negative maximum error, the correlation relation and the standard deviation For the statistical comparison of different equations, the Model of Selection Criterion and the Akaike Information Criterion were used.	It was found that the Zigrang and Sylvester equation gives the most precise value of friction factor and that the Haaland equation is more adequate for manual calculations.
Brkić (2011b)	This article shows a review of the existing explicit approximations comparisons with the Colebrook-White equation.	The majority of the available approximations of the Colebrook-White equation, are very precise. The exceptions are the approximations made by Round, Eck, Moody, Wood, Rao and Kumar. The mean error of nearly all the explicit approximations of Colebrook-White relation is up to 3%.
Salmasi *et al*. (2012)	The performance of the explicit formulations for the friction factor and the techniques of artificial intelligence (AI) are studied. The AI techniques used include artificial neuronal nets (ANN) and genetic programming (GP). Tests included Re and ε/D transformations, using a logarithmic scale.	This study shows that some of the explicit formulations for the friction factor induce to improper errors, but some of them have god precision The ANN formulation for the resolution of the friction factor is less successful than the GP formulation.

Winning and Coole (2013)	28 explicit equations for the friction factor are revised, their precision with respect to implicit Colebrook-White equation and their relative calculation efficiency are analyzed. For the revision, ranges of Reynolds numbers 4.103 $\leq$ Re $\leq$ 4.10^8 and pipeline relative roughness $10^{-6} \leq \varepsilon/D \leq 10^{-1}$, were selected.	2D and 3D contour graphics were generated which show the range and magnitude of the relative precision percent in the whole range of points for each explicit equation.
Dobrnjac (2012)	Precision and complexity of 15 explicit approximations to the Colebrook-White equation for the determination of the friction factor were studied. The maximum relative error for each approximation was determined and the complexity was valued.	It is demonstrated that the approximations obtained by adjusting Moody Diagram made with C-W formula and Nickuradse measurements, are not successful in the transition flow zone and cover only the turbulent flow above Re=4000. Studies that have eliminated these problems and determine a function of commutation formula for the friction coefficient for all Reynolds numbers Re ($0 \leq \mathrm{Re} \leq 10^8$) and all relative roughness values, are described. This formula is more precise than C-W formula and then all the other previously published equations..
Mohsenabadi *et al*. (2014)	Report of 29 explicit relations from different researchers which are proved in precision.	It is confirmed that the relations of Serghides (1984), Vatankhah and Kochakzadeh (2008) and Buzzelli (2008), with a relative error below $\pm$ 0.05 % for different Reynolds numbers and relative roughness, give the best results. Goudar and Sonnad (2008) and Avci and Karagoz (2009), show the highest level of error in comparison to C-W relation.
Asker *et al*. (2014)	Various correlations of friction factors are revised. The relative error is evaluated for different Re values and relative pipeline roughness. Statistical analysis is done for each correlation.	It was found that in some of this correlations the percent error is so small that they can be used directly in place of the Colebrook-White equation.
Anaya *et al*. (2014)	Various mathematical models that describe explicitly the friction factor with respect to Colebrook-White equation and the Kármán number were evaluated. The evaluation was made for a relative roughness value of (ε/D=0.001) and Reynolds numbers between 4.103 and 10^8.	The use of Pavlov (Frenkel) correlation is recommended.

Lipovka and Lipovka (2014)	A comparative analysis was made of existing formulas for the Darcy's friction factor in turbulent regime and Colebrook-White resolved by the Clamon[1] method. The absolute mean square deviation is shown in graphics	It is concluded that the Clamond method gives the major precision for all the ranges of ε/D. The second place is occupied by the Goudar and Sonnad equation.
Offor and Alabi (2016b)	The performances of explicit models not based in AI (artificial intelligence) were compared with performances of models based in AI.	The genetic algorithm optimizes the model´s explicit parameters, with improvements of 0.12 % to 0.0026 %, according to the index of maximum relative error. Although the genetic programming produces explicit analytical formulas for the determination of output parameters, it was found that they are extremely inaccurate with errors up to 7 % in some models.(Lipovka & Lipovka, 2014)
Lukman and Oke (2018)	Precision of explicit friction factor formulas were evaluated using the relative error, the selection criterion model (SCM) and the Akaike information model (AIM) and were compared with the Colebrook-White formula for Re between entre 4.10^{-3} y $1{,}704.10^{8}$ y ε/D entre 10^{-7} y 0.052, using Microsoft Excel -Solver.	Formulas with lowest relative error are: Shaikh *et al*. (0.03%); Serghides (0.70 %); Buzzelli (0.70 %); Vatankhah and Kouchakzadeh (0.71 %); Romeo *et al*. (0.71 %); Sounnad and Goudar (0.71 %); Swamee and Jain (0.73%); Barr and White (0.73 %); Manadilli (0.73 %); Churchill (0.74 %); Fang *et al*. (0.75 %); Chen (0.76 %); Barr and White (0.77%); Evangelids *et al*. (0.80 %); Zigrang and Sylvester (0.84 %); Eck (0.86 %); Brikic (0.93 %); Jain (0.86 %); Haaland (1.52 %); Wood (3.48 %); Ghanbari *et al*. (2.17 %).
Pimenta *et al*. (2018)	29 explicit equations found in literature were analyzed and λ was determined by means of Re in the range of $4.103 \leq \text{Re} \leq 10^{8}$ and relative roughness (ε/D) of $10^{-6} \leq \varepsilon/D \leq 5.10^{-2}$. The performance index and the relative error of formulations were analyzed in relation to the Colebrook-White equation.	The analysis found 7 equations with an excellent performance and high precision, standing out the Offor and Alabi formulation, which can be used as alternative to the standard Colebrook-White equation.

[1]The Clamond method is a special algorithm of iterative calculation of λ, which gives accuracy close to limits of computer type double after two iterations. It requires calculation of logarithm once for initial estimation and one time per iteration. (Lipovka and Lipovka, 2014)

The Akaike's information criterion (AIC) is a measure of the relative quality of a statistical model for a set of data. The AIC provides a medium for the model selection. AIC manages a balance between the model's adjustment goodness and the model's complexity. According to this criterion, the most appropriate model is the one that has the smaller value of the AIC. Meanwhile, the model of selection criterion (MSC), is derived from the Akaike's information criterion and allows a direct comparison between models with a different number of parameters (PN). For this criterion, the most appropriate model is the MSC, bigger, as it propitiates maximizing the model's content of information (Romeo, Royo & Monzón, 2002); while the MSE provides a good indication of the global maximum percent precision of the explicit equation. The peaks in the data are only visualized with the error.

Table 6. Precision of the analyzed formulas as reported by previous research.

Eq. [#]	Author	Relative mean error [%]	Relative máximum error [%]	Computational efficiency (Winning y Coole, 2013)
1	Moody (1947)	4.76-7.52	21.5	2
2	Altshul (1963)		16.5*	
3	Altshul (1963)	11.45-16.42	21.5*	1
4	Agroskin (1954)		2.3*	
5	Frenkel (1956)		2.8*	
6	Lobaev (1956)		31.0*	
7	Chernikin (1958)		4.75*	
8	Wood (1966)	3.65-3.88	23.8	3
9	Churchill (1973)	0.08-0.48	2.2	7
10	Eck (1966)	1.50-2.23	8.2	4
11	Jain (1976)	0.18-0.86	2	10
12	Swamee and Jain (1976)	0.04-0.93	2	8
13	Churchill (1977)	0.74	2.2	11
14	Chen (1979)	0.07	0.4	19
15	Round (1980)	3.71-4.47	10.9	6
16	Schorle *et al*. (1980)	0.13-0.32	0.8	13
17	Barr and White (1981)	0.06	0.3	21
18	Zigrang and Sylvester (1982)	0.00061-0.84	1	15
19	Zigrang and Sylvester (1982)	0.02	0.1	23
20	Haaland (1983)	0.21	1.4	14
21	Serghides (1984)	0.06	0.1	20
22	Serghides (1984)	0.04	0.4	17
23	Tsal (1989)	8.89-16.16	37.5*	5
24	Robaina (1992)	0.84	2.4*	
25	Manadilli (1997)	0.03-0.74	2.1	18
27	Sousa *et al*. (1999)		0.1*	
28	Romeo *et al*. (2002)	0.06	0.1	27
29	Dobromyslov (2004)		10.7	
30	Sonnad and Goudar (2006)	0.17	0.8	9
31	Rao and Kumar (2007)	13.27-16.15	82	24
32	Buzzelli (2008)	0.005	0.1	25
33	Vantankhah and Kouchakzadeh (2008)	0.36-0.71	7.6*	8
34	Avci and Karagoz (2009)	1.18-1.72	4.7	12
35	Papaevangelou *et al*. (2010)	0.23	0.9	28
36	Chernikin and Talipov (2010)		46.0*	
37	Brkić (2011a)	0.12-0.72	2.3	26
38	Brkić (2011a)	0.12-0.48	2.3	22
39	Fang *et al*. (2011)	0.06	0.4	16
40	Ghanbari *et al*. (2011)	2.17	3.0*	
41	Samadianfard (2012)	0.08	7	
42	Shaikh *et al*. (2015)	0.03	10.2	
43	Brkić (2016)	0.49-3.87	8.9*	
44	Offor and Alabi (2016a)	0.0025	0.2*	

Eq. [#]	Author	Relative mean error [%]	Relative máximum error [%]	Computational efficiency (Winning y Coole, 2013)
45	Beluco and Schettini (2016)		4.3*	
48	Azizi *et al*. (2018)		1.6*	

Source: (11, 23, 29, 34, 47, 65, Authors) *For the ranges of Reynolds numbers and relative roughness declared in the table 4.

Observation: Expressions 26, 46 and 47 are not included for being valid for all flow regimes.

Some of the explicit equations have very high errors in a small range of input values, generating peaks that, although being obvious in graphics, could not be so easily identified in data tables and, certainly, cannot be recognized if MSE and maximum relative percent error, have not been reported (Winning and Coole, 2013). These criteria were used by Genić *et al*. (Jaric *et al*., 2011) and Yildirim (2009) in the comparison of various explicit models.

Brkić (2011a) based on the maximum relative percent error criterion, classified existing explicit models as: extremely precise: with error ≤ 0.14%, very precise: error up to 0.5%, moderately precise: error up to 1.5%, less precise: error up to 5%, not recommended: error up to 25% and extremely inexact: error ≥ 80%.

The first review on the precision of λ explicit formulations was done in April of 1985 by Gregory and Fogarasi (1985). A second review was made in June of the same year by Zigrang and Sylvester (1985), using the same boundary conditions as used by Serghides (1984). This study was very similar to the one made by Gregory and Fogarasy in relation to the explicit equations revised, with the exception of the rejection of the Wood equation (Wood,1966) and the inclusion of Chen equation (Chen,1979). Goudar and Sonnad (2007, 2008) made other two revisions.

Table 5 shows the main research works done in the last 10 years, on precision and complexity of the different explicit expressions. It is necessary to point out that, all the work of review on the precision of explicit equations for the calculation of the friction factor, has been done for extremely broad ranges of Reynolds numbers and relative roughness, which are not found in common practice.

Yildirim (2009) made another study based in the same matrix of values used by Goudar and Sonnad. Brkić (2011b) made a revision of 20 equations and in the same year another revision of 16 equations in the same range of values as the ones of Goudar and Sonnad and Yildirim, was made. These studies were made with a very large matrix of a million points, which introduced the Altshul equation, cited by Jaric *et al*. (2011). This revision, as the work of Romeo *et al*. (2002) used the Selection Criteria Model (SCM) and the Akaike Information Criteria (AIC), to make a statistical comparison of the relative computational efficiency that concluded in the recommendation of using Zigrang and Sylvester (1982) equation. Beluco and Schettini (2016) analyzed six classic approximations of Colebrook-White equation and they also propose a generic model for this category, which results equally complex for reiterative calculations in the solution of fluid mechanics problems. Jaric *et al*. (2011) stated that the Zigrang and Sylvester equation (Zigrang and Sylvester, 1982) gives the most accurate value of the friction factor and that the Haaland (1983) equation, with similar complexity as the Swamee and Jain equation (1976), is the most appropriate for manual calculations. Pimenta *et al*. (2018) conclude that the equations (Chen, 1979) and those of Sonnad and Goudar (2006), Buzzelli (2008), Vantankhah and Kouchakzadeh (2008), Fang *et al*. (2011) and Offor and Alabi (2016b), have high precision in comparison to the Colebrook-White approximation.

Table 6 shows the mean and maximum relative error of the expressions reported by different authors cited in previous reviews, in respect to Colebrook-White equation.

Table 6 also shows the calculated errors for the formulas reported for the first time by this review and other recalculated values in which there is no agreement with previous reports or had not been reported (equations 2, 3, 4, 5, 6, 7, 23, 24, 27, 33, 36, 40, 44 and 45).

From the analysis made, which is shown in Table 6, three tendencies in the development of these expressions can be observed: the first one, is the use of algebraic equations (1, 3, 8, 26, 36, 41); the second one, is the use of expressions that include base 10 logarithm -the most abundant- (equations: 2, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 31, 35, 40, 42, 43, 45, 48); and the third one, those which use the natural logarithm (equations: 13, 30, 32, 33, 34, 37, 38, 39, 44, 46, 47).

Table 7. Equations with relative errors below 5% for turbulent flow in pipelines with relative roughness ($10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$) and Reynolds numbers ($10^{-6} \leq \varepsilon/D \leq 5\times10^{-2}$), ordered according to their precision.

Order number	Equation number	Author (Reference)	Equation
1	27	Sousa and col. (1999)	$\frac{1}{\sqrt{\lambda}} = -2log\left[\frac{\varepsilon}{3.7D} - \frac{5.16}{Re}log\left(\frac{\varepsilon}{3.7D} + \frac{5.09}{Re^{0.87}}\right)\right]$
2	22	Serghides (1984)	$\lambda = \left[4.781 - \frac{(A_7 - 4.781)^2}{A_8 - 2A_7 + 4.781}\right]^{-2}$ $A_7 = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{12}{Re}\right)$ $A_8 = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51A_7}{Re}\right)$
3	30	Sonnad and Goudar (2006)	$\frac{1}{\sqrt{\lambda}} = 0.8686ln\left(\frac{0.4587Re}{G^{(G/G+1)}}\right)$ $G = 0.1240\times\frac{\varepsilon}{D}\times Re + \ln(0.4587Re)$
4	20	Haaland (1983)	$\lambda = -1.8log\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)^{-2}$
5	48	Azizi and col. (2018)	$\lambda = \left[1.805log\left(\frac{\left(\frac{\varepsilon}{D}\right)^{1.108}}{4.267} + \frac{5.164}{Re^{0.966}}\right)\right]^{-2}$
6	9	Churchill (1973)	$\frac{1}{\sqrt{\lambda}} = -2log\left[\frac{\varepsilon}{3.7D} + \left(\frac{7}{Re}\right)^{0.9}\right]$
7	4	Agroskin (1954)	$\frac{1}{\sqrt{\lambda}} = -1.8.log\left[\frac{6.8}{Re} + \left(\frac{\varepsilon}{3.7D}\right)^{1,1}\right].$
8	5	Frenkel (1956)	$\frac{1}{\sqrt{\lambda}} = -2.log\left(\left(\frac{6.81}{Re}\right)^{0,9} + \frac{\varepsilon}{3.7D}\right)$
9	12*	Swamee and Jain (1976)	$\lambda = \frac{0.25}{\left[log\left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$
10	34	Avci and Karagoz (2009)	$\lambda = \frac{6.4}{\left\{ln(Re) - ln\left[1 + 0.01Re\frac{\varepsilon}{D}\left(1 + 10\sqrt{\frac{\varepsilon}{D}}\right)\right]\right\}^{2.4}}$
11	7	Chernikin (1958)	$\frac{1}{\sqrt{\lambda}} = -1.83.\,log\left[\frac{8.5}{Re} + \left(\frac{\varepsilon}{3.7D}\right)^{1.093}\right]$

* The Swamme-Jain equation shows a maximum relative error of 3.12% in this range, for the rest of the equations, it is shown in Table 6.

Table 8. Valid equations (relative errors less than 5%) for turbulent flow in smooth pipelines or pipelines with low relative roughness ($0 \le \varepsilon/D \le 5\times10^{-2}$) and Reynolds numbers ($4\times10^3 \le Re \le 10^8$), ordered according to their precision.

Order number	Equation number	Author (Reference)	Equation
1	28	Romeo and col. (2002)	$\frac{1}{\sqrt{\lambda}} = -2log\left\{\frac{\varepsilon}{3.7065D} - \frac{5.0272}{Re}log\left[\frac{\varepsilon}{3.827D} - \frac{4.567}{Re}log(A)\right]\right\}$ $A = \left(\frac{\varepsilon}{7.7918D}\right)^{0.9924} + \left(\frac{5.3326}{208.815Re}\right)^{0.9345}$
2	32	Buzzelli (2008)	$\frac{1}{\sqrt{\lambda}} = B_1 - \left[\frac{B_1 + 2log\left(\frac{B_2}{Re}\right)}{1+\frac{2.18}{B_2}}\right]$ $B_1 = \frac{[0.774ln(Re)] - 1.41}{\left(1+1.32\sqrt{\frac{\varepsilon}{D}}\right)}$ $B_2 = \frac{\varepsilon}{3.7D}Re + 2.51B_1$
3	44	Offor and Alabi (2016a)	$\frac{1}{\sqrt{\lambda}} = -2log\left\{\frac{\varepsilon}{3.71D} - \frac{1.975}{Re}\left[ln\left(\left(\frac{\varepsilon}{3.93D}\right)^{1.092} + \left(\frac{7.627}{Re+395.9}\right)\right)\right]\right\}$
4	39	Fang and col. (2011)	$\lambda = 1.613\left\{ln\left[0.234\left(\frac{\varepsilon}{D}\right)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right]\right\}^{-2}$
5	16	Schorle and col. (1980)	$\frac{1}{\sqrt{\lambda}} = -2log\left[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}log\left(\frac{\varepsilon}{3.7D} + \frac{14.5}{Re}\right)\right]$
6	25	Manadilli (1997)	$\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{\varepsilon}{3.7D} + \frac{95}{Re^{0.983}} + \frac{96.82}{Re}\right)$
7	37	Brkić (2011a)	$\frac{1}{\sqrt{\lambda}} = -2log\left(10^{-0.4343\beta} + \frac{\varepsilon}{3.71D}\right)$ $\beta = ln\frac{Re}{1.816ln\left[\frac{1.1Re}{ln(1+1.1Re)}\right]}$
8	38	Brkić (2011a)	$\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{2.18\beta}{Re} + \frac{\varepsilon}{3.71D}\right)$
			$\beta = ln\frac{Re}{1.816ln\left[\frac{1.1Re}{ln(1+1.1Re)}\right]}$
9	40	Ghanbari and col. (2011)	$\lambda = \left\{-1.52log\left[\left(\frac{\varepsilon}{7.21D}\right)^{1.042} + \left(\frac{2.731}{Re}\right)^{0.9152}\right]\right\}^{-2.169}$
10	45	Beluco and Schettini (2016)	$\lambda = \frac{0.3009}{\left[log\left(\left(\frac{\varepsilon}{3.7315D}\right)^{1.0954} + \left(\frac{5.9802}{Re}\right)^{0.9695}\right)\right]^2}$

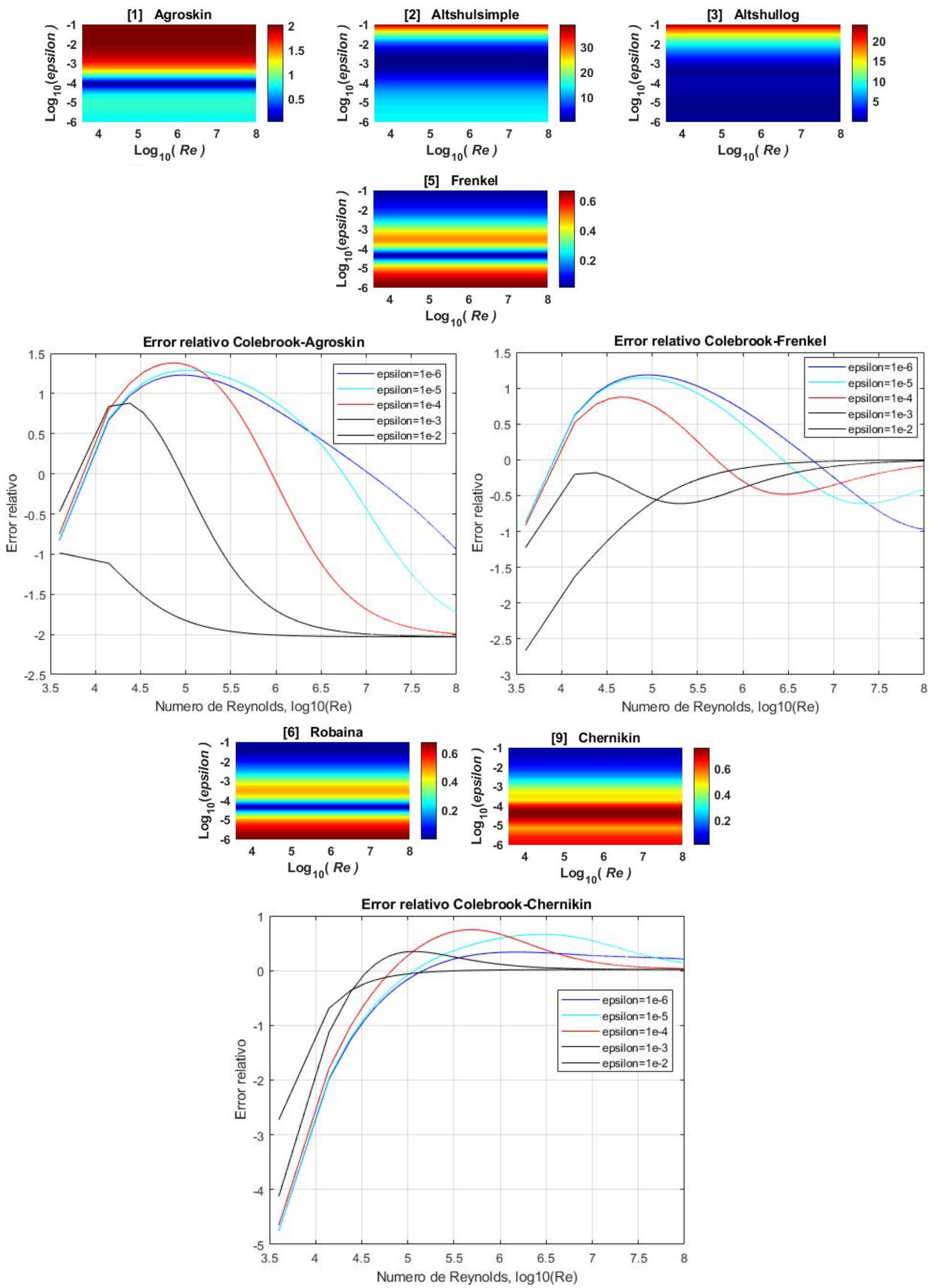

Fig. 2. Relative error of explicit expressions selected in accordance to the Colebrook equation (épsilon representa el valor de ε/D).

In table 7 appear the 11 equations that have relative errors lower than 5% valid for turbulent flow in rough pipelines ($10^{-6} \leq \varepsilon/D \leq 5 \times 10^{-2}$) and Reynolds numbers in the range $4 \times 10^3 \leq Re \leq 10^8$, which have been ordered according to their precision. Equations of Sousa *et al*. (1999), Serghides (1984) and Sonnad and Goudar (2006), have a relative error lower than 1% and are the most precise ones; among these, the equation of Sousa *et al*. (1999), is the less complex one.

Meanwhile, for smooth pipelines or pipelines with low relative roughness ($0 \leq \varepsilon/D \leq 5 \times 10^{-2}$), there are 10 equations with relative errors lower than 5% which are valid for turbulent flow and Reynolds numbers ($4{\times}10^3 \leq Re \leq 10^8$) (table 8). The equations of Romeo *et al*. (2002), Buzzelli (2008), Offor and Alabi (2016a), Fang *et al*. (2011) y Schorle *et al*. (1980), have a relative error lower than 1% and are the most precise ones; but nevertheless, they are quite complex, except the Schorle *et al*. (1980) equation, whose complexity is relatively low.

The main conclusion on the precision of explicit expressions for the calculation of hydraulic friction coefficient in turbulent flow is, that the relative error is not uniformly distributed in the Reynold number´s domain (Re) and the relative roughness (ε/D) (Brkić and Ćojbašić, 2017). In figure 2 the formulas of Agroskin (Eq. 4), Frenkel (Eq. 5), Altshul simple (Eq. 3) and Altshul logarithmic (Eq. 2), result vey imprecise for high values of relative roughness; but nevertheless, for relative roughness values lower than $1{\times}10^{-2}$, the two first formulas have significantly lower error and give acceptable results for engineering calculations.

On the other hand, the behavior of equations of Chernikin (Eq. 7) and Robaina (Eq. 24), is different, because the major values of relative error are observed in pipelines with small values of relative roughness (Fig. 2).

The great majority of the most recognized authors in fluid mechanics, do not agree in the recommendation of an equation for the calculation of λ in turbulent regime. Çengel and Cimbala (2017) state that for the explicit approximate relation given by Haaland (1983), results are in the 2% of the ones obtained by the Colebrook-White equation and less than 1% with respect to the equation of Churchill (1977).

Mott and Untener (2015) consider the Swamee and Jain (1976) equation, as it allows the direct calculation of λ value for turbulent flow and gives values that are in ±1% into the range of relative roughness: $\varepsilon/d = 0.01$ a 1.10^{-6} and for Reynolds numbers of 5.10^3 a 10^8. This is virtually all the turbulent zone of the Moody diagram. The authors state that, although the equations of Agroskin (4), Frenkel (5) and Robaina (24), have similar complexity to the equations of Haaland y Swamee-Jain , which are the most frequently used in Western countries for practical calculations; their precision, lower to these last ones, does not merit its use. Equations of Sousa *et al*. (27) and Offor and Alabi (44), are very precise, but their use is not recommended owed to their complexity. Because of its simplicity, the possible use of the simple algebraic equation of Altshul (Eq. 3), should be evaluated for the common ranges that are used in practical cases, although for wide ranges of Reynolds numbers and relative roughness, it shows a high imprecision.

Conclusions

1. In this study the explicit equations proposed by different researchers, to approximate the values obtained by the Colebrook-White implicit equation were identified and compiled, as well as the precision reported in the literature and the relative computational efficiency of more than 45 explicit relations, and it was confirmed that there is discrepancy in the data given by different authors about the precision and complexity of these equations. From these results, it is recommended to use the equations of Haaland and Swamee-Jain, because of their acceptable precision and medium complexity.

2. In this research, the relative maximum error of various explicit equations not included in previous reports, was determined and the same error of other equations that had been studied previously, was adjusted. It was confirmed once again that, the higher the precision in comparison to the Colebrook-White equation, higher will be the complexity of the expressions. Likewise, the relative error varies for the different values of relative roughness and Reynolds numbers.

3. The work developed on the explicit equations precision has been done for an extremely wide range of Reynolds numbers and relative roughness, which are not of common use in the engineers practice and in the study of Fluid Mechanics, so it is necessary to determine the

precision of explicit equations for a smaller range of Re and relative roughness, and on this base, propose which of these equations could be more adequate to be used in practical cases.

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Nomenclature

Re: Reynolds Number, dimensionless
D: Interior pipeline diameter, m

Greek symbols
λ: Friction factor or Friction Coefficient, dimensionless
ε Pipeline absolute roughness, m

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