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Fundamentals in the design and scaling of biodigesters with mixing by hydraulic recirculation of wastewater for biogas production using dimensional analysis

Fundamentos en el diseño y escalamiento de biodigestores con mezclado por recirculación hidráulica de agua residual para la producción de biogás utilizando análisis dimensional

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Abstract

In the present work, fundamental relationships and criteria were established for the characterization and scaling of biogas production using a biodigester operated with mixing by hydraulic recirculation of wastewater. A biodigester with a cylindrical-conical combined geometry was selected and, using the scale factor and biodigester dimensional relationships, equations were developed for its geometrically scaling, building a program in *Simulink* (*Matlab*® software) denominated *biodigester.slx*. Seventeen process variables were identified and were correlated through dimensional analysis according to the π theorem of Vaschy-Buckingham, to generate the dimensionless numbers of Geometry, Reynolds, Power, Recirculation, and Damköhler I. The dimensionless functions and dimensionless numbers allowed the development of expressions for power consumption and the biogas production volumetric rate from the recirculation velocity and hydraulic recirculation rate, respectively. The above was carried out considering the following three aspects: the behavior of the wastewater as a non-Newtonian fluid, the sludge and manure concentration, and the mixing effect by hydraulic recirculation of the wastewater. Furthermore, from the geometric, kinematic, dynamic, and chemical similitude principles, criteria and equations were established for scaling of recirculation velocity, power consumption, and biogas production volumetric rate. The work is a platform to study biogas production by anaerobic digestion in biodigesters operated by batch, with mixing by hydraulic recirculation of wastewater.

Keywords: Biogas production, anaerobic digestion, biodigester, hydraulic recirculation, scaling.

Resumen

En el presente trabajo se establecieron las relaciones fundamentales y los criterios para la caracterización y escalamiento de la producción de biogás en un biodigestor operado con mezclado por recirculación hidráulica del agua residual. Un biodigestor con una geometría combinada cilíndrica-cónica fue seleccionado, y con el factor de escala y las relaciones dimensionales del biodigestor se desarrollaron las ecuaciones para su escalamiento geométrico, construyendo un programa en *Simulink* (software *Matlab*®) denominado *biodigester.slx*. Se identificaron 17 variables de proceso y mediante análisis dimensional acorde con el teorema π de Vaschy-Buckingham fueron correlacionadas para generar los números adimensionales de Geometría, Reynolds, Potencia, Recirculación y Damköhler I. Las funciones adimensionales y los números adimensionales permitieron desarrollar expresiones para el consumo de energía y la tasa volumétrica de producción de biogás a partir de la velocidad de recirculación y la tasa de recirculación hidráulica, respectivamente. Lo anterior se realizó considerando los siguientes tres aspectos, el comportamiento de las aguas residuales como fluido no newtoniano, la concentración de lodos y estiércol, y el efecto del mezclado por recirculación hidráulica del agua residual. Además, a partir de los principios de similitud de geometría, cinemática, dinámica y química, se establecieron criterios y ecuaciones para escalar la velocidad de recirculación hidráulica, el consumo de energía y la tasa volumétrica de producción de biogás. El trabajo es una plataforma para estudiar la producción de biogás por digestión anaerobia en biodigestores operados por lote con mezclado por recirculación hidráulica del agua residual.

Palabras clave: Producción biogas, digestión anaerobia, biodigestor, recirculación hidráulica, escalamiento.

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1 Introduction

The energy demand at a global level is increasing significantly, and around 88% is covered by fossil fuels (Michaelowa *et al*., 2018; Christophers, 2019). This high energetic dependence on fossil fuels has also caused an environmentally negative impact, both by pollution (Hernández-Martínez *et al*., 2019; Sandoval-Herazo *et al*., 2020) and the greenhouse effect. The above has led to implementing technologies for the use of clean energies, allowing diminished dependence on carbon and petroleum in the medium term (IEA, 2015). Biogas generation by anaerobic digestion is within these technologies (Nishio *et al*., 2007). Anaerobic digestion is a biological process that has been used for decades as an alternative in wastewater treatment and all the microbiological aspect of it are known (Cabezas *et al*., 2015; Mao *et al*., 2015). Anaerobic digestion has become relevant because of its potential to convert several kinds of soluble waste to biogas, which is integrated principally by methane, carbon dioxide, hydrogen, and hydrogen sulfide. This new approach is recognized as having a double advantage since, besides decreasing wastewater, pollution also allows access to a low-cost energy source (Achinas *et al*., 2017).

Biogas generation by anaerobic digestion from high charges of total solids (>10%) has been demonstrated to be more suitable in biodigesters operated by batch and has been conducted at the laboratory scale (0.25 to 40 L) (Zhang *et al*., 2008; Guendouz *et al*., 2010; Anjos *et al*., 2017; Lee *et al*., 2019) and pilot plant scale (60 to 280 L) (Anozie *et al*., 2005; Di Maria *et al*., 2013; Onofre *et al*., 2015). The development of this technology has been focused principally on two aspects: firstly, in the feasibility of biogas generation, depending on the kind of waste substrate present in the wastewater (Mata-Alvarez *et al*., 2000; Zhu *et al*., 2011; Teniza-García *et al*., 2015); and secondly, in the design and operational conditions of the biodigesters (Zupancic *et al*., 2008; Kaparaju *et al*., 2009; Nkemka *et al*., 2010; Liu *et al*., 2012). In this last aspect, the mixing of biodigesters has been shown to have a significant effect on biogas generation due to improving the mass transfer, giving a homogeneous environment to bacteria (Monteith *et al*., 1981), enhancing the anaerobic digestion process and, hence, the production of biogas. Studies of the mixing effect on the anaerobic digestion to produce biogas has been conducted in three different systems: stirred tanks with mechanical mixing (Kaparaju *et al*., 2008; Ghanimeh *et al*., 2012; Kowalczyk *et al*., 2013), biogas recirculation (Demirer *et al*., 2005; Yu *et al*., 2010), and the hydraulic recirculation of the wastewater (Karim *et al*., 2005b; Karim *et al*., 2008; Xiong *et al*., 2015). In stirred tanks with mixing by mechanic impellers, the efficiency in power consumption is recognized by volume unity. However, the energy requirements with their growing size increase by several orders of magnitude. This energetically compromises the scaling of these kinds of biodigesters (Karim *et al*., 2005c). In the mixing by biogas recirculation, there are two strong constraints to their use. The first constraint is the necessary instrumentation that must be implemented to avoid air infiltration, and with this, the anaerobic digestion inhibition. The second restriction is because of the low mixing efficiency at a high concentration of anaerobic sludge (Karim *et al*., 2005a). In the hydraulic recirculation of wastewater made up of manure and sludges at 10% by weight, biogas production levels have been higher in comparison with mixing by mechanical agitation (31%) and biogas recirculation (93%) using biodigesters with the same size and shape (Karim *et al*., 2005b). Furthermore, Xiong *et al*. (2015) determined that biogas generation from cattle manure was 18.4% greater in a biodigester with wastewater recirculation that in a biodigester without mixing. Thus, the biodigesters with mixing by hydraulic recirculation of wastewater have been shown to be a feasible alternative in biogas generation by anaerobic digestion. Nevertheless, to improve yield and increase the biogas generation in biodigesters with wastewater recirculation, the anaerobic sludge and organic waste charge must be higher than 10% (Rico *et al*., 2011). On the other hand, to our knowledge, there is no information that establishes the fundamentals or principles in design and scaling of biodigesters with hydraulic recirculation of wastewater for biogas production. The scope of this information must consider diverse aspects such as biodigester shape, dimensional criteria for their geometric scaling, and the determination of the mixing regime associated with rheological properties of wastewater, as well as the power consumption, biogas generation dependence of the mixing regime, and the concentration of the anaerobic sludges together with the organic matter in the wastewater.

Scaling is an indispensable tool in the engineering area since it establishes criteria relating to the change of scale in equipment, systems or processes, maintaining their characteristics and properties

(Dudukovic *et al*., 2015; López *et al*., 2018). In its more simple conception, scaling is the step of one size to another for the development of products, equipment, or technologies. When the change of size is of laboratory level to another upper level (pilot or industrial), it is denominated "scale-up", whereas, if the change is towards a smaller size, it is denominated "scale down". Scaling is supported by the similarity principle, since this allows the establishment of geometric, mechanic, thermic, and chemical similitudes that must be satisfied to have accurate scaling (Zlokarnik, 2006). Within biogas generation by anaerobic digestion, there are reports about scaling of some of the stages, from pretreatments to biodigesters (Svensson *et al*., 2006; Rittman *et al*., 2008; Li *et al*., 2017). However, these are focused only on biodigesters without mixing. Therefore, it is necessary to develop criteria accord to similarity principles for the scaling of biodigesters with mixing by hydraulic recirculation of wastewater. The aim of this work was to establish, by means of dimensional analysis and similitude principles, the fundamentals for the development of criteria, mathematical relationships, and expressions that can be used as a platform in the design and scaling of biodigesters with mixing by hydraulic recirculation of wastewater for production of biogas.

2 Materials and methods

Development of the fundamentals, criteria, and mathematical relationships for the design and scaling of biodigesters with mixing by hydraulic recirculation of wastewater for biogas production was supported in the following strategy:

i) Adequate geometry of the biodigester for the mixing and biogas generation was established. With the scale factor and mathematical relationships among the dimensions of the biodigester, expressions for their geometrical scaling were developed. Furthermore, a program in *Matlab®-Simulink* was constructed to determine the geometric dimensions of the scaled biodigester.

ii) Dimensional analysis of the process from the π theorem of Vaschy-Buckingham was conducted, which allowed generation of the dimensionless numbers that correlate the identified variables of biogas production in the biodigester with mixing by hydraulic recirculation of the wastewater.

iii) With the dimensionless functions and dimensionless numbers, expressions for the design and operating conditions were established for biogas production in the biodigester with mixing by hydraulic recirculation. Aspects such as the characterization of biogas generation through the process variables were discussed. In characterization, the rheological properties of wastewater as a non-Newtonian fluid, as well as the concentration of the anaerobic sludge and organic waste on the wastewater, were taking into account.

iv) Scaling criteria were established, based on the geometric, kinematic, dynamic, and chemical similitude principles. These criteria allowed the development of expressions for scaling the hydraulic recirculation velocity, power consumption, and volumetric biogas production. The development of each one of the strategy steps mentioned above is presented in the Results and Discussion section.

3 Results and discussion

3.1 Geometrical scaling of the biodigester

The cylindrical and rectangular geometries are the two principals used in the biodigester design due to them being straightforward (Sánchez-Rubal, 2016). Nevertheless, these geometries have serious problems in the accumulation of settleable solids and in the mixing. On the other hand, biodigesters with a conical bottom have demonstrated improved mixing (Karim *et al*., 2007). Hence, one biodigester with a combined cylindric-conic geometry was selected for biogas production with mixing by hydraulic recirculation of wastewater. Figure 1 shows the geometry and dimensional relationships of the biodigester. The diameter of the cylindric middle section (D_c) was selected as the characteristic dimension for conducting the geometrical scaling. The nominal volume of the biodigester (V_{biod}) can be estimated from the geometries of each one of the cylindric and segmented conic sections according to the equation (1).

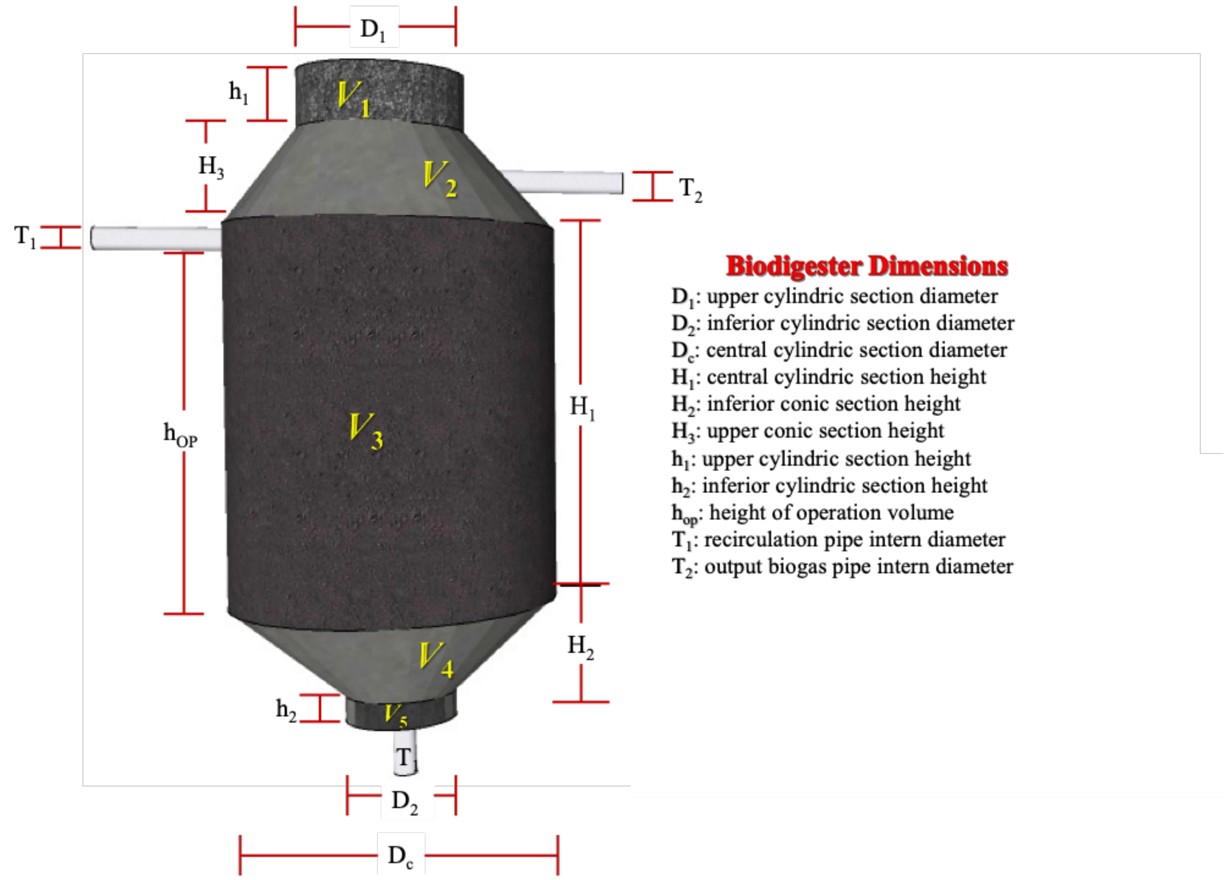

Fig. 1. Dimensional relationships and geometry of the biodigester proposed in the biogas production by hydraulic recirculation of wastewater.

$$V_{biod} = \frac{\pi}{4}(D_1^2 \cdot h_1 + D_c^2 \cdot H_1 + D_2^2 \cdot h_2) + \frac{\pi}{12}\left[H_3(D_c^2 + D_1^2 + D_c \cdot D_1) + H_2(D_c^2 + D_2^2 + D_c \cdot D_2)\right] \tag{1}$$

Using D_c as the geometric base and taking account of internal diameters of the recirculation pipe (T_1) and the biogas output pipe (T_2), nine geometric constants can be defined (K_1, K_2, K_3, k_1, k_2, d_1, d_2, t_1, t_2) which are given by the equations (2)-(4).

$$\frac{H_1}{D_c} = K_1; \quad \frac{H_2}{D_c} = K_2; \quad \frac{H_3}{D_c} = K_3 \tag{2}$$

$$\frac{h_1}{D_c} = k_1; \quad \frac{h_2}{D_c} = k_2; \quad \frac{H_{op}}{D_c} = K_1 - t_1 \tag{3}$$

$$\frac{D_1}{D_c} = d_1; \quad \frac{D_2}{D_c} = d_2; \quad \frac{T_1}{D_c} = t_1; \quad \frac{T_2}{D_c} = t_2 \tag{4}$$

Putting the biodigester dimensions in equations (2) to (4) in terms of geometric constants and D_c, an expression is obtained for the nominal volume, as given by the equation (5).

$$V_{biod} = \frac{\pi}{4} D_c^3 \left\{ \left(K_1 + d_1^2 \cdot k_1 + d_2^2 \cdot k_2\right) + \frac{1}{3}\left[K_3(d_1^2 + d_1 + 1) + K_2(d_2^2 + d_2 + 1)\right] \right\} \tag{5}$$

The operation volume (wastewater volume inside the biodigester, V_{op}) is determined from the volumes V_3, V_4, V_5, and the distance with respect to the bottom of the recirculation pipe of the biodigester output ($h_{op} = H_1 - T_1$).

$$V_{op} = \frac{\pi}{4} D_c^3 \left\{ \left(K_1 - t_1 + d_2^2 \cdot k_2\right) + \frac{1}{3}[K_2(d_2^2 + d_2 + 1)] \right\} \tag{6}$$

$$D_c* = \sqrt[3]{\frac{4 \cdot N \cdot V_{biod}}{\pi\{(K_1 + d_1^2 \cdot k_1 + d_2^2 \cdot k_2) + \frac{1}{3}[K_3(d_1^2 + d_1 + 1) + K_2(d_2^2 + d^2 + 1)]\}}} \quad (7)$$

The equations (2)-(6) represent the dimensions, and the nominal and operation volumes of the biodigester in their original scale. According to the original biodigester dimensions and the nominal volume, for a scaling factor "N", the new dimensions and nominal volume of the scaled biodigester are given by ($V_{biod}^* = N * V_{biod}$, where the new dimensions of the scaled biodigester are also identified with an asterisk):

$$H_1* = K_1 \cdot D_c*;\ \ H_2* = K_2 \cdot D_c*;\ \ H_3* = K_3 \cdot D_c* \quad (8)$$

$$h_1* = k_1 \cdot D_c*;\ \ h_2* = k_2 \cdot D_c*;\ \ h_{op}* = H_1 * - T_1* \quad (9)$$

$$D_1* = d_1 \cdot D_c*;\ D_2* = d_2 \cdot D_c*;\ T_1* = t_1 \cdot D_c*;\ T_2* = t_2 \cdot D_c* \quad (10)$$

The equations (7)-(10) work for a scale-up ($N > 1$) as well as a scale-down ($N < 1$). These sets of equations were used to estimate, in a scale factor of one hundred, the scale-up (N=100) and scale-down (N=1/100) of a commercial biodigester of the Rotoplas trademark, which has a cylindrical-conical combined geometry as indicated in Figure 1. The commercial Rotoplas biodigester volume is 788 L (V_{biod}=0.788 m^3) and the dimensions in meters are: $H_1 = 0.94$; $H_2 = 0.36$; $H_3 = 0.35$; $h_1 = 0.1$; $h_2 = 0.035$; $D_1 = 0.55$; $D_2 = 0.24$; $D_c = 0.85$; $T_1 = 0.08$; $T_2 = 0.1$; $h_{op} = H_1 - T_1$. The estimations were conducted with a program developed in *Simulink* of *Matlab* 12.1, which was denominated *biodigester.slx*. The program requires only the introduction of the original biodigester dimensions in meters and the scale factor (N).

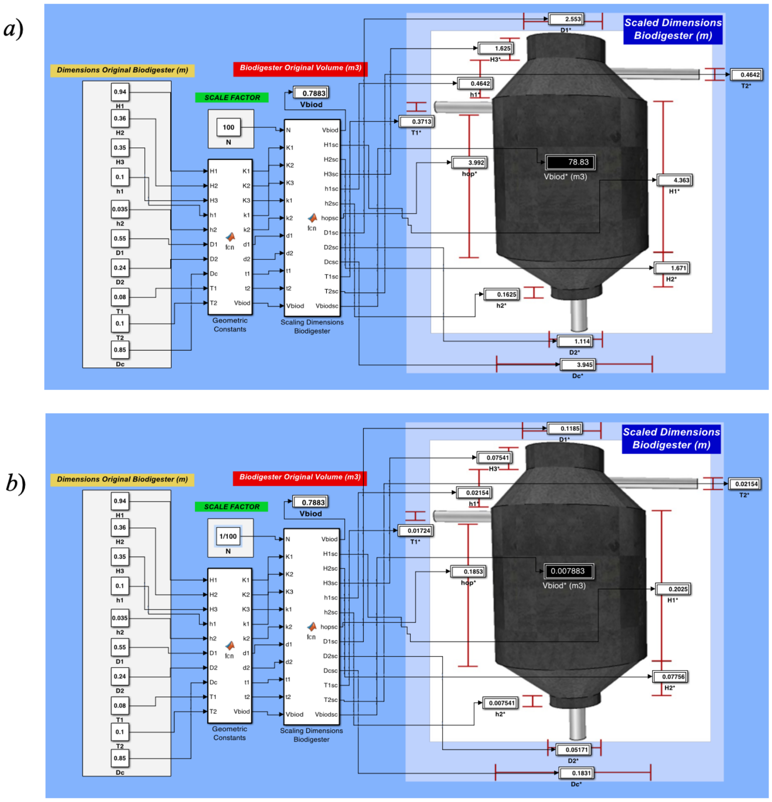

Fig. 2. Geometry scaling in a factor of one hundred of the commercial ovoid biodigester (trademark Rotoplas): a) *scale-up* (N=100); b) *scale-down* (N=1/100). The simulation was carried out with *biodigester.slx* program of *Simulink, Matlab* 8.3.

Figure 2 shows results generated by the program *biodigester.slx* in the scale-up, as well as in the scale-down, of a biodigester by a factor of one hundred with respect to its original size. Although scale changes were 100-fold up and down in volume, the characteristic dimension D_c only changed by around 21.5-fold in both directions since linear variation in scale change to a lesser extent than volume variation.

3.2 Dimensional analysis of biogas production in a biodigester with mixing by hydraulic recirculation of wastewater

In order to determine how process variables in biogas generation in a biodigester with mixing by hydraulic recirculation of wastewater can be related, dimensional analysis was conducted. Figure 3 shows the scheme of the biodigester with the seventeen identified process variables, while Table 1 lists and groups the variables based on their properties. Moreover, Table 1 also indicates the derived magnitudes of each process variable according to the fundamental magnitudes of longitude (L), mass (M), and time (t). According to the π theorem of Vaschy-Buckingham, it is assumed that there is a dimensional function that describes the biogas generation in the biodigester with mixing by hydraulic recirculation of wastewater which must be integrated by the seventeen process variables (N_{vp} = 17).

$$f(v_f, P, \eta, \rho, R_{bio}, \tau, D_c, D_1, D_2, H_1, H_2, H_3, h_1, h_2, h_{op}, T_1, T_2) = 0 \tag{11}$$

The function given by equation (11) can be described dimensionally by the three fundamental magnitudes ($N_{mf} = 3; L, M, t$); therefore, the number of dimensionless groups (Π) that describe the process is $\Pi = N_{vp} - N_{mf} = 14$. These fourteen dimensionless groups $\pi_i (i = 1, 2, \ldots, 14)$ conform to the equivalent dimensionless function given by equation (12).

$$F(\pi_1, \pi_2, \ldots, \pi_{13}, \pi_{14}) = 0 \tag{12}$$

According to the π theorem, each dimensionless group must be integrated by $R = 3$ fixed variables and one that will have to be isolated (Var_i). In this work, the variables D_c, v_f, and ρ_{ap} were selected as fixed for the construction of the groups π_i. These variables are representative of one property of the process, such as geometric, kinematic, and of the wastewater as fluid, respectively. Table 2 shows the dimensional relations of the fixed selected variables for the construction of groups π_i. The determinant of the matrix (I) formed by the dimensional relations of the fixed selected variables was used to evaluate the independence between them.

Table 1. Definition, symbol and magnitudes of process variables identified on the biogas generation in a biodigester with mixing by wastewater recirculation.

Variable	Symbol	Magnitude
Kinematic and Dynamic properties		
Wastewater recirculation velocity	v_f	$L\,t^{-1}$
Power consumption	P	$M\,L^{2}t^{-3}$
Properties of wastewater as fluid		
Apparent viscosity	η_{ap}	$M\,L^{-1}\,t^{-1}$
Apparent density	ρ_{ap}	$M{\cdot}L^{-3}$
Bioreaction properties		
Biogas volumetric generation rate	R_{bio}	$M\,L^{-3}\,t^{-1}$
Recirculation rate	R_{rc}	t^{-1}
Geometric properties		
Central cylindric section diameter	D_c	L
Upper cylindric section diameter	D_1	L
Inferior cylindric section diameter	D_2	L
Central cylindric section height	H_1	L
Inferior conic section height	H_2	L
Upper conic section height	H_3	L
Upper cylindric section height	h_1	L
Inferior cylindric section height	h_2	L
Height of operation volume	h_{op}	L
Recirculation pipe intern diameter	T_1	L
Output biogas pipe intern diameter	T_2	L

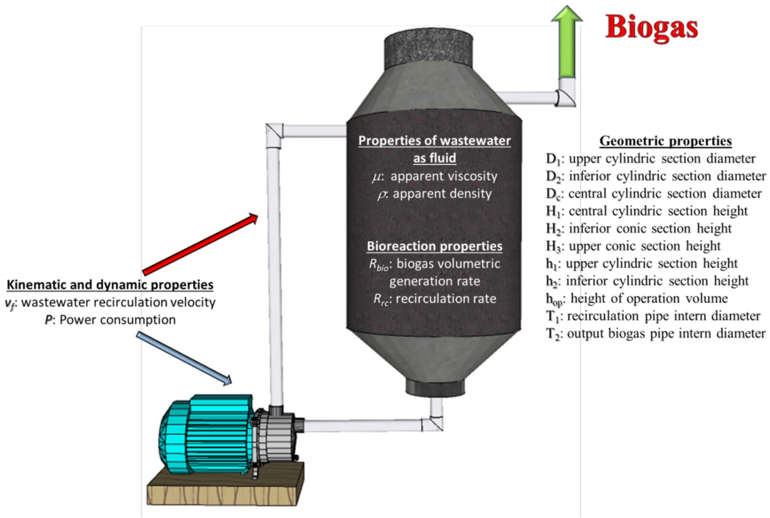

Fig. 3. Process variables identified on biogas generation in a biodigester with mixing by hydraulic recirculation of wastewater.

Table 2. Dimensional relations of the fixed selected variables for the groups π_i construction.

Fundamental magnitude	v_f	ρ_{ap}	D_c
L	1	-3	1
M	0	1	0
t	-1	0	0

$$\det(I) = \begin{vmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 0$$

The variables are independent since their determinant is different to zero and, therefore, can be used to construct the groups π_i, whose structure will have the following general form:

$$\pi_i = v_f^{a_{i1}} \cdot \rho_{ap}^{1_{i3}} \cdot Var_i = 1 = M^0 \cdot L^0 \cdot t^0 \qquad (13)$$

Next, the groups π_i can be constructed, corresponding to each one of the variables to isolate. Beginning with the *power consumption* due to hydraulic recirculation of the wastewater (P), the dimensionless form of this first group π is:

$$\pi_1 = v_f^{a_{11}} \cdot \rho_{ap}^{a_{12}} \cdot D_c^{a_{13}} \cdot P = 1 = M^0 \cdot L^0 \cdot t^0 \qquad (14)$$

Substituting the derivate magnitude of each variable:

$$\begin{aligned} \pi_1 &= (Lt^{-1})^{a_{11}} \cdot (ML^{-3})^{a_{12}} \cdot (L)^{a_{13}} \cdot (ML^2t^{-3}) \\ &= 1 = M^0 \cdot L^0 \cdot t^0 \end{aligned} \qquad (15)$$

gives the equation system:

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$$

whose solution is $a_{11} = -3$; $a_{12} = -1$; $a_{13} = -2$. The dimensionless group π_1 corresponds to the *Power Number* (N_P).

$$N_p = \frac{P}{v_f^3 \cdot \rho_{ap} \cdot D_c^2} \qquad (16)$$

N_P considers the relationship between the external force keeping the wastewater moving inside the biodigester due to hydraulic recirculation of the wastewater and the inertial forces by unit volume on the inside of the biodigester.

The next variable to isolate is the apparent viscosity of the wastewater (η_{ap}), whose dimensionless group it is given by:

$$\pi_2 = v_f^{a_{21}} \cdot \rho_{ap}^{a_{22}} \cdot D_c^{a_{23}} \cdot \eta_{ap} = 1 = M^0 \cdot L^0 \cdot t^0 \qquad (17)$$

Substituting the corresponding derivate magnitudes of each variable:

$$\begin{aligned} \pi_2 &= (Lt^{-1})^{a_{21}} \cdot (ML^{-3})^{a_{22}} \cdot (L)^{a_{23}} \cdot (ML^{-1}t^{-1}) \\ &= 1 = M^0 \cdot L^0 \cdot t^0 \end{aligned} \qquad (18)$$

gives the equation system:

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

whose solution is $a_{21} = -3$; $a_{22} = -1$; $a_{23} = -2$. The inverse of the dimensionless group π_2 corresponds to the *Reynolds Number* (N_{Re}).

$$N_{Re} = \frac{v_f \cdot \rho_{ap} \cdot D_c}{\eta_{ap}} \tag{19}$$

The N_{Re} considers the relationship of the inertial force to move the wastewater inside the biodigester with respect to viscous forces. An apparent viscosity (η_{ap}) is contemplated in this N_{Re} because the wastewater (anaerobic sludge with cattle manure) behaves as a non-Newtonian fluid, being dependent of the shear velocity. The N_{Re} of equation (19) will be modified posteriorly according to the model considered to describe the apparent viscosity as a function of shear velocity.

For the *biogas volumetric generation rate*, the dimensionless relationships is given by:

$$\pi_3 = v_f^{a_{31}} \cdot \rho_{ap}^{a_{32}} \cdot D_c^{a_{33}} \cdot R_{bio} = 1 = M^0 \cdot L^0 \cdot t^0 \tag{20}$$

Substituting the corresponding derivate magnitudes of each variable:

$$\begin{aligned} \pi_3 &= (Lt^{-1})^{a_{31}} \cdot (ML^{-3})^{a_{32}} \cdot (L)^{a_{33}} \cdot (ML^{-3}t^{-1}) \\ &= 1 = M^0 \cdot L^0 \cdot t^0 \end{aligned} \tag{21}$$

gives the equations system:

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

whose solution is $a_{31} = -1$; $a_{32} = -1$; $a_{33} = 1$. The dimensionless group π_3 corresponds to the *Damköhler I Number* ($N_{Da\mathrm{I}}$).

$$N_{Da\mathrm{I}} = \frac{R_{bio} \cdot D_c}{v_f \cdot \rho_{ap}} \tag{22}$$

In the biogas generation by wastewater recirculation in the biodigester, $N_{Da\mathrm{I}}$ considers the relationship between bioreaction time ($\tau_{bio} = \rho_{ap}/R_{bio}$) and mixing time ($\tau_m = D_c/v_f$), where $N_{Da\mathrm{I}}=\tau_m/\tau_{bio}$.

The next variable to isolate is the *recirculation rate*, whose dimensionless group is given by:

$$\pi_4 = v_f^{a_{41}} \cdot \rho_{ap}^{a_{42}} \cdot D_c^{a_{43}} \cdot R_{rc} = 1 = M^0 \cdot L^0 \cdot t^0 \tag{23}$$

Substituting the corresponding derivate magnitudes of each variable:

$$\begin{aligned} \pi_4 &= (Lt)^{a_{41}} \cdot (ML^{-3})^{a_{42}} \cdot (L)^{a_{43}} \cdot (t^{-1}) \\ &= 1 = M^0 \cdot L^0 \cdot t^0 \end{aligned} \tag{24}$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{41} \\ a_{42} \\ a_{43} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

whose solution is $a_{41} = -1$; $a_{42} = 0$; $a_{43} = 1$. The dimensionless group π_4 corresponds to the *Recirculation Number* (N_r).

$$N_r = \frac{R_{rc} \cdot D_c}{v_f} \tag{25}$$

The N_r considers the relationship between wastewater interchange time within the biodigester ($\tau = 1/R_{rc}$) and mixing time (τ_m), where $N_r = \tau_m/\tau$. The N_r has been used to study the recirculation effect on the mixing patterns of biodigesters with mediums that simulate the density and rheological properties of wastewater by means of the addition of xanthan gum (Low *et al*., 2017).

Designating the geometric dimensions D_1, D_2, H_1, H_2, H_3, h_1, h_2, h_{op}, T_1, T_2 as G_i, each can be isolated in the same way on dimensionless groups with the following structure:

$$\pi_{Gi} = v_f^{a_{Gi1}} \cdot \rho_{ap}^{a_{Gi2}} \cdot D_c^{a_{Gi3}} \cdot G_i = 1 = M^0 \cdot L^0 \cdot t^0 \tag{26}$$

Substituting the corresponding derivate magnitudes of each variable:

$$\begin{aligned} \pi_{Gi} &= (Lt^{-1})^{a_{Gi1}} \cdot (ML^{-3})^{a_{Gi2}} \cdot (L)^{a_{Gi3}} \cdot (L) \\ &= 1 = M^0 \cdot L^0 \cdot t^0 \end{aligned} \tag{27}$$

gives the equations system:

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

whose solution is $a_{G1} = 0$; $a_{G2} = 0$; $a_{G3} = -1$. The dimensionless groups π_{Gi} correspond to the geometric constants given in the equations (2)-(4) and are defined as *Geometry Numbers* (N_{Gi}).

$$N_{Gi} = \frac{G_i}{D_c} \tag{28}$$

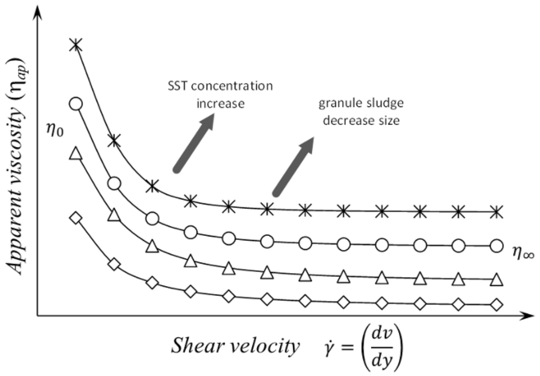

Fig. 4. Typical rheogram for apparent viscosity (η_{ap}) characterization on the observed wastewater behavior like a non-Newtonian fluid.

3.3 Derivation of fundamental relations for the characterization and design of the biodigester with mixing by hydraulic recirculation of wastewater for biogas production

The dimensional analysis generated five dimensionless numbers that correlate the seventeen process variables identified in the biogas generation using a biodigester with mixing by hydraulic recirculation of wastewater. These dimensionless numbers define the dimensionless function given by:

$$F(N_{G1}, \ldots N_{G10}, N_{Re}, N_P, N_r, N_{Da\mathrm{I}}) = 0 \tag{29}$$

The dimensionless function allows different fundamental relations to be proposed between dimensionless numbers obtained by the dimensional analysis. A first fundamental relation can be established by recognizing that power consumption for recirculation depends on biodigester geometry, wastewater recirculation velocity, and wastewater properties such as the apparent viscosity and density. Therefore, there is a sub-domain of the dimensionless function described by the explicit function given in equation (30) that will contemplate the *Power Number* (N_P), Reynolds Number (N_{Re}), and Geometry number (N_{Gi}).

$$F_{\mathrm{I}}(N_{G1}, \ldots N_{G10}, N_{Re}, N_P) = 0 \tag{30}$$

From equation (30), an implicit function can be determined which establishes a functional relationship between N_P, N_{Re}, and N_{Gi}, in which the dimensionless numbers can be related by a potential expression given by:

$$N_p = \Phi(N_{G1}, \ldots, N_{G10}, N_{Re}) = k_p \cdot N_{G1} \cdot \ldots \cdot N_{G10} \cdot N_{Re}^m \tag{31}$$

where k_p and m are the characteristic constant and the characteristic potency exponent, respectively. In the explicit function given by equation (31) the Geometry Numbers and the characteristic constant can be grouped in a new constant denominate K_{Gp}, obtaining the next potential expression:

$$N_p = K_{Gp} \cdot N_{Re}^m \tag{32}$$

The power consumption (P) in the biodigester with mixing by hydraulic recirculation of wastewater can be characterized experimentally with equation (32). However, before describing how to carry out the characterization, the nature of the Reynolds Number in equation (32) should be considered. In this case, the Reynolds Number corresponds to a non-Newtonian fluid since the wastewater, composed of anaerobic sludge and organic matter like cattle manure, has a rheological behavior where apparent viscosity is a function of shear velocity ($\dot{\gamma}$) (Liu *et al.*, 2016). In this wastewater, the apparent viscosity also depends on the total suspended solid concentration and granular sludge size (Eshtiaghi *et al.*, 2013). Therefore, it is necessary to first have a characterization of the apparent wastewater viscosity to determine a Reynolds Number expression. The apparent wastewater viscosity (η_{ap}) can be characterized based on its rheological behavior through experimental rheogram construction (Figure 4) (Ayol *et al.*, 2006; Baudez *et al.*, 2011; Eshtiaghi *et al.*, 2012). Two of the principal models used to fit and describe the wastewater apparent viscosity dependence on the velocity shear are the Ostwald de Waele power-law model given by equation (33) and the Sisko model given by equation (34) (Liu *et al.*, 2016; Eshtiaghi *et al.*, 2013).

$$\eta_{ap} = K \cdot \dot{\gamma}^{(n-1)} \tag{33}$$

$$\frac{\eta_{ap} - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1 + K \cdot \dot{\gamma}^n} \tag{34}$$

In both models, K and n are the consistency and behavior indexes, respectively, while in the Sisko model, η_0 and η_∞ are the viscosities of the wastewater with respect to low and high shear velocities, respectively. In the present work, the Ostwald de Waele power-law model will be used. In this model, the linear regression of experimental data of the natural logarithm of shear velocity versus the natural logarithm of apparent viscosity allows us to determine the values of the parameters of K and n from the interception with the Y axis and the slope. Substituting the power-law model in the Reynolds Number defined by equation (19) and recognizing that the shear velocity inside of the biodigester is given by $\dot{\gamma} = v_f/D_c$, the next expression for the Reynolds Number is obtained.

$$N_{Re} = \frac{v_f^{2-n} \cdot D_c^n \cdot \rho_{ap}}{K} \tag{35}$$

The Reynolds Number obtained is similar to that reported by Metzner *et al.* (1995) for non-Newtonian fluids in pipes with the internal diameter as a characteristic dimension. In our study, D_c is the characteristic dimension. Once the Reynolds Number for the wastewater as a non-Newtonian fluid has been established, and knowing the K and n parameters, it is possible to characterize the power consumption by hydraulic recirculation of wastewater with equation (32). The experimental power consumption (P_{exp}) for different recirculation velocities (v_f) of the wastewater can be measured using a wattmeter connected online with the circuit of the pump engine. In turn, v_f can be determined in terms of the volumetric flow and the internal diameter of the recirculation pipe.

$$v_f = \frac{4 \cdot V_g}{\pi \cdot T_1^2} \tag{36}$$

With the different P_{exp} values in the function of the v_f , the Power Number and Reynolds Number can be estimated with equations (16) and (35), and the K_{Gp} and m parameter values, in turn, could be determined from equation (33). It should be noted that m and K_{Gp} determination will be dependent on the mixing regime. In the laminar regime, there is a linear relationship between the Power Number and the Reynolds Number, so that the K_{Gp} and m parameters can be determined by the linear regression of the natural logarithms of N_P versus N_{Re}. In the transient regime, the parameters could be determined by methods of non-linear regression while, for the turbulent regime, the Power Number is probably independent of the Reynolds number.

An expression for power consumed due to mixing by hydraulic recirculation of wastewater in the biodigester can be derived from equations (16), (32) and (35), given by:

$$P = \frac{K_{Gp}}{K^m} \cdot \rho_{ap}^{m+1} \cdot D_c^{mn+2} \cdot v_f^{m[2-n]+3} \tag{37}$$

The power consumption in the mixing by hydraulic recirculation of wastewater in biodigesters has been determined theoretically by $P = \rho \cdot g \cdot H_v \cdot V_g$ (ρ: medium density (m^3/kg),$\cdot H_v$: hydraulic head (m), g: gravity (m/s^2) and V_g: volumetric flow (m^3/s)), which considers the wastewater both as a Newtonian and a non-Newtonian fluid (Karim *et al.*, 2005b; Low *et al.*, 2017). In the Computational Fluid Dynamic (CFD) (Wu *et al.*, 2008; Terashima *et al.*, 2009), this equation has also been used to simulate the non-Newtonian fluid behavior of anaerobic sludges. The theoretical estimation of the power consumption using the above mentioned equation is suitable for Newtonian fluids. Nevertheless, in the case of non-Newtonian fluids as wastewater, it is not suitable because of the variation of apparent viscosity with shear velocity. In this aspect, equation (37) takes account of the variation of viscosity; moreover, it contemplates parameters of design and operation, being more representative in the power consumption theoretical estimation.

A second relationship between dimensionless numbers can be generated by recognizing that the biogas production volumetric rate depends of the biodigester geometry, wastewater properties, and the recirculation rate. The recirculation rate affects the biogas production volumetric rate due to the mixing level (regime) associated with the replacement time of the wastewater within the biodigester. According to the above, it can be assumed that the Damköhler I number ($N_{Da\text{I}}$) is related with the Geometry and Recirculation numbers (N_{Gi} and N_r) in an explicit function defined in a subdomain of equation (29), which is given by:

$$N_{Da\text{I}} = \Psi(N_{G1}, \ldots, N_{G10}, N_r) \tag{38}$$

Equation (38) establishes a dimensionless relationship between $N_{Da\text{I}}$, N_{Gi}, and N_r, proposing a potential function in a similar form to that of the Power and Reynolds numbers.

$$N_{Da\text{I}} = k_r \cdot N_{G1} \cdot \ldots \cdot N_{G10} \cdot N_r^q \tag{39}$$

In equation (39), k_r and q are the characteristic constant and the characteristic potency exponent, respectively. These are parameters associated with explicitly describing the function given by equation (38). Grouping the Geometry Numbers and characteristic constant in a new constant K_{Gr}, the next potential expression is obtained:

$$N_{Da\text{I}} = K_{Gr} \cdot N_r^q \tag{40}$$

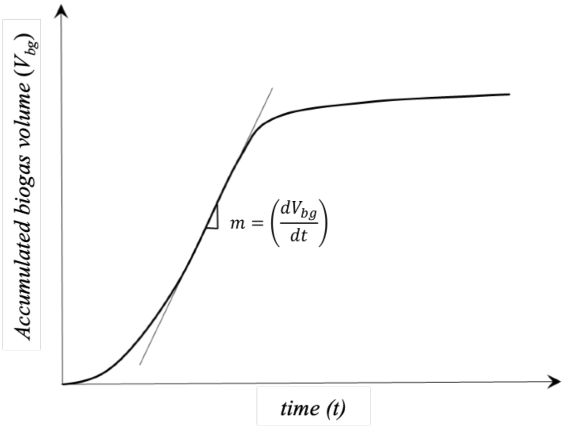

Fig. 5. Determination of the biogas volumetric rate from accumulated biogas volume in the batch at a given recirculation velocity.

From equation (40), the biogas production volumetric rate (R_{bio}) can be characterized with regard to the recirculation rate (R_{rc}). Taking into account that R_{rc} is the inverse of the replacement time of the wastewater within the biodigester, the recirculation rate can be estimated from recirculation velocity by:

$$R_{rc} = \frac{v_f \cdot \pi \cdot T_1^2}{4 \cdot V_{op}} \tag{41}$$

For a given recirculation rate (R_{rc}), an apparent density of the wastewater (ρ_{ap}), and biodigester geometry (D_c y T_1), the experimental biogas production volumetric rate (R_{bioexp}) can be evaluated from the slope of the accumulated biogas volume in the batch (dV_{bg}/dt) (Figure 5). Considering that the biogas behavior is that of an ideal gas, the experimental biogas production volumetric rate is estimated by:

$$R_{bioexp} = \frac{p \cdot PM_{bg}}{R \cdot T_{op} \cdot V_{op}} \cdot \left(\frac{dV_{bg}}{dt}\right) \tag{42}$$

where p is the atmospheric pressure, R the ideal gas constant, T_{op} the operational temperature, PM_{bg} the molecular weight of the biogas, and V_{bg} the biogas accumulated volume. With the values of R_{bioexp} determined at the different R_{rc}, and the dimensionless numbers of Damköhler 1 and Recirculation, the values of the parameters q and K_{Gr} can be estimated from equation (40). Nevertheless, the method for determining both parameters will depend on the mixing regime in which the experimental process has been conducted. Knowing the q and K_{Gr} values, an expression for the biogas production volumetric rate can be derived from equations (22), (25) and (40), which is given by:

$$R_{bio} = K_{Gr} \cdot \rho_{ap} \cdot R_{rc}^q \left[\frac{D_c}{v_f}\right]^{q-1} \tag{43}$$

The apparent density has as relevant a part to play as both the biogas production volumetric rate and the power consumption, since this property is associated with the concentration of anaerobic sludge and organic matter in the wastewater. It is clear that increases in the concentration of anaerobic sludge and organic matter lead to improvements in the biogas production, but also increase the energy required for the hydraulic recirculation of the wastewater. Hence, a proper determination of this property is necessary. Landry *et al.* (2004) showed that, for concentrations of anaerobic sludge and organic matter of less than 30% in weight, the apparent density of the weighted form could be estimated by:

$$\rho_{ap} = (1 - f_{ld}) \cdot \rho + f_{ld} \cdot \rho_{ld} \tag{44}$$

where ρ_{ld} and f_{ld} are the density and weight fraction of the anaerobic sludge and organic matter in the wastewater, respectively. Landry *et al.* (2004) also showed that, for concentrations above 50% in weight, the apparent density does not follow linear behavior with regard to weight fraction, which is probably due to their nature as a non-Newtonian fluid. For high concentrations of the anaerobic sludge with organic matter, Landry *et al.* fitted the wastewater density to a polynomial of third grade:

$$\rho_{ap} = a_0 + a_1 \cdot f_{ld} + a_2 \cdot f_{ld}^2 + a_3 \cdot f_{ld}^3 \tag{45}$$

The power consumption (equation (37)) and the biogas production volumetric rate (equation (43)) are in terms of the recirculation velocity as the operation variable. An increase in the recirculation velocity leads to an increase in the power consumption but also increases the level of mixing, improving the biogas production volumetric rate. Nevertheless, a high mixing intensity could have a negative effect on the biogas production rate. In the evaluation of the mixing effect on biogas production by anaerobic digestion, both at laboratory level (Kaparaju *et al.*, 2008) and in the biodigester (Lindmark *et al.*, 2014b), it has been determined that a moderate mixing level improves the biogas production, while a strong mixing level leads to a decrease in production, even to levels lower than those obtained without mixing.

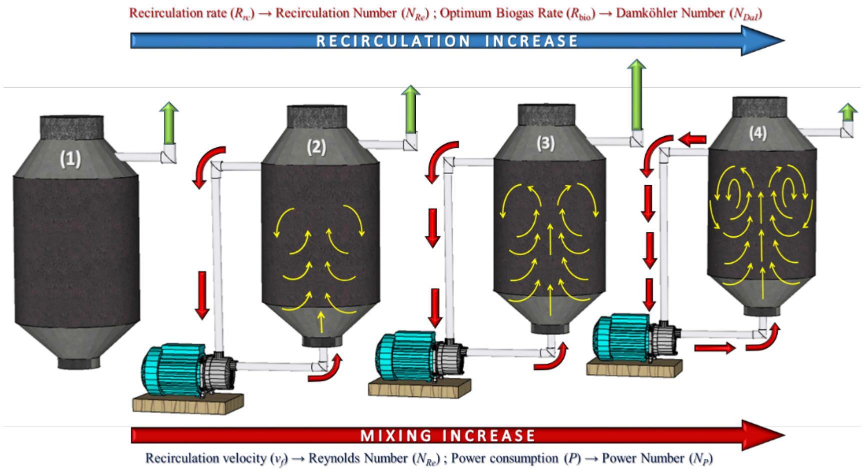

Fig. 6. Scheme of the mixing effect on biogas production by hydraulic recirculation of wastewater.

The negative effect of an intense mixing can be associated with the rupture of the granules of the anaerobic sludge due to the shear force that modifies both the extracellular substances production and the equilibrium between the acetogenic and methanogenic microbial population, leading to acidification of the medium (Lindmark *et al.*, 2014a). According to the above, there is an optimum in the mixing level regime for biogas production. Figure 6 schematizes the effect of mixing on biogas production by the hydraulic recirculation of wastewater. The expressions for the power consumption and biogas production volumetric rate established by means of the dimensional analysis in this work, in a batch process with hydraulic recirculation of wastewater in a biodigester with cylindrical-conical combined geometry, will allow determination of the energetic feasibility of the process through experimental characterization.

3.4 Scaling of biogas production with mixing by hydraulic recirculation of wastewater

In the scaling of the process, it is necessary to comply with the similitude principles. The present work considers an isothermal process by batch and with recirculation; hence, the similitude principles to cover are geometric, kinetic, dynamic, and chemical. The geometric similitude principle was covered with the mathematical relationships established for the geometric constants defined by the equations (2)-(4).

The kinetic similitude principle is directly related to the Reynolds number, which considers the mixing regime within the biodigester due to recirculation velocity. To maintain the kinetic similitude, the N_{Re} must be the same in both scales, therefore:

$$N_{Re1} = N_{Re2} \tag{46}$$

where N_{Re1} and N_{Re2} are the Reynolds numbers in the original scale and the new scale, respectively. Substituting the corresponding Reynolds number for each scale according to equation (35) gives:

$$\frac{v_{f1}^{2-n} \cdot D_{c1}^{n} \cdot \rho_{ap}}{K} = \frac{v_{f2}^{2-n} \cdot D_{c2}^{n} \cdot \rho_{ap}}{K} \tag{47}$$

Considering that the apparent density and the consistency index in both scales are the same, the recirculation velocity of the wastewater in the new scale of the biodigester is given by:

$$v_{f2} = \left(\frac{D_{c1}}{D_{c2}}\right)^{\frac{n}{2-n}} \cdot v_{f1} \tag{48}$$

In equation (48), the scaling of the recirculation velocity varies in a non-linear form with respect to the quotient of the characteristic diameters between the original and new scales. This is by way of the dependency with regard to index n, due to the wastewater behaving as a non-Newtonian fluid. For wastewater, the index n is reported with values between zero and one, with values decreasing when the concentration of sludge increases (Cao *et al.*, 2016; Khalili-Garakani *et al.*, 2011; Achkari-Begdouri, 1992). It should be noted that as the sludge concentration decreases, then $n \rightarrow 1$, $n/(2-n) \rightarrow 1$, and the wastewater behavior tends to a Newtonian fluid. From equation (48), it can be predicted that the recirculation velocity decreases in a scale-up and increases in a scale-down.

The dynamic similitude principle is related to the Power Number (N_P), which contemplates the energy flow required to maintain the mixing regime. The dynamic similitude is conserved when the Power Numbers between scales are the same.

$$N_{P1} = N_{P2} \tag{49}$$

where N_{P1} and N_{P2} are the Power Numbers on the original scale and the new scale, respectively. Substituting for each scale, the corresponding Power Numbers according to equation (16) are:

$$\frac{P1}{v_{f1}^3 \cdot \rho_{ap} \cdot D_{c1}^2} = \frac{P_2}{v_{f2}^3 \cdot \rho_{ap} \cdot D_{c2}^2} \tag{50}$$

In both scales, the apparent density is the same. Substituting equation (48) into equation (50) and placing in terms of the power consumption of the original scale (P_1):

$$P_2 = \left(\frac{D_{c1}}{D_{c2}}\right)^{\frac{5n-4}{2-n}} \cdot P_1 \tag{51}$$

From equation (37), for the original scale in equation (51), the following is obtained:

$$P_2 = \frac{K_{Gp}}{K^m} \cdot \rho_{ap}^{m+1} \cdot \left[\frac{v_{f1}^{\frac{2-n}{n}} \cdot D_{c1}}{D_{c2}^{\frac{5n-4}{\alpha}}}\right]^{\frac{\alpha}{2-n}} \tag{52}$$

where $\alpha = n \cdot [m \cdot (2-n) + 3]$. Equation (52) estimates the power consumption for the new scale in terms of amounts such as the wastewater properties (ρ_{ap}, K, n), the characteristics diameters (D_{c1} and D_{c2}) of the biodigester geometry trough, the recirculation velocity in the original scale (v_{f1}) and the design parameters (K_{Gp} and m) associated with the power number dependence on the Reynolds number.

The last similitude principle is chemical, which is associated with the Damköhler I number ($N_{Da\mathrm{I}}$) which contemplates the biogas production volumetric rate under the mixing regime due to the recirculation velocity of the wastewater within the biodigester. The chemical similitude it is conserved when the Damköhler I numbers between the two scales are the same.

$$N_{Da\mathrm{I}1} = N_{Da\mathrm{I}2} \tag{53}$$

where $N_{Da\mathrm{I}1}$ and $N_{Da\mathrm{I}2}$ are the Damköhler I numbers in the original scale and the new scale, respectively. Substituting the corresponding power numbers for each scale according to equation (22) gives:

$$\frac{R_{bio1} \cdot D_{c1}}{v_{f1} \cdot \rho_{ap}} = \frac{R_{bio2} \cdot D_{c2}}{v_{f2} \cdot \rho_{ap}} \tag{54}$$

In both scales, the apparent density is the same. Substituting equation (48) into equation (54) and placing in terms of the biogas production volumetric rate of the original scale (R_{bio1}):

$$R_{bio2} = \left(\frac{D_{c1}}{D_{c2}}\right)^{\frac{2}{2-n}} \cdot R_{bio1} \tag{55}$$

From equation (43), for the original scale in equation (55), the following is obtained:

$$R_{bio2} = K_{Gr} \cdot \rho_{ap} \cdot R_{rc1}^q \left[\frac{D_{c1}^{\beta}}{v_{f1}^{\beta+1} \cdot D_{c2}^2}\right]^{\frac{1}{2-n}} \tag{56}$$

where $\beta = \cdot[(2-n)q + n]$. Equation (59) estimates the biogas production volumetric rate in the new scale (R_{bio2}) in terms of amounts such as wastewater properties (ρ_{ap}, K, n), the characteristics diameters (D_{c1} and D_{c2}) of the biodigester geometry trough, the recirculation velocity in the original scale (v_{f1}), and the design parameters (K_{Gr} and q) associated with the Damköhler I dependence on the Recirculation number.

The equations established from the similitude principles and the dimensionless numbers of Reynolds, Power, Recirculation, and Damköhler I are an approach of design for the scaling of the biogas production from a biodigester mixing by hydraulic recirculation of wastewater composed of a mixture of anaerobic sludge and manure. In bioreactor scaling, the scale factor values most used are among two and three magnitude orders (Löffelholz *et al.*, 2013). However, the scaling of the processes decreases in the accuracy as the scale factor size is increased (Ju and Chase, 1992). Hence, it is advisable to use the mathematical relationships developed in the present work with a scale factor no greater than 150.

Conclusions

The mathematical relationships with criteria for design and scaling of biogas production from anaerobic digestion in a biodigester operated by batch and with mixing by hydraulic recirculation of wastewater were established. One biodigester with a cylindrical-conical combined geometry was selected as the most suitable for the biogas production process by hydraulic recirculation of wastewater. The scaling equations of the biodigester were established with the scale factor and the geometrical relationships. A program in *Simulink-Matlab* denominated *biodigester.slx* was developed for conducting the calculations. In the biogas production process, 17 variables were identified, and through the dimensional analysis supported in the π de Vaschy-Buckingham theorem, the Geometry, Reynolds, Power, Recirculation, and Damköhler I dimensionless numbers were generated. From the dimensionless functions, correlations for the dimensionless numbers were obtained, allowing established mathematical relationships for the power consumption and biogas production volumetric rate, which can be experimentally characterized through wastewater properties as a non-Newtonian fluid and the correlations between dimensionless numbers through the recirculation velocity. Finally, mathematical relationships for the scaling of the recirculation velocity, power consumption, and biogas production volumetric rate were established from dimensionless numbers and similitude principles. The present work is a platform that establishes the fundamentals for the design and scaling of biogas production from anaerobic digestion in biodigesters operated by batch with a cylindrical-conical combined geometry and mixing by hydraulic recirculation of wastewater.

Acknowledgements

In memory of Doctor Mariano Gutiérrez Rojas, who was our professor, colleague, and friend, teaching us to love the profession of engineering and also to appreciate and enjoy the field of biotechnology.

Nomenclature

a	Second parameter of the Khalili-Garakani model, $M^{1-b}\ L^{3b-1}\ \mathrm{t}^{-2}$
b	First parameter of the Khalili-Garakani model, dimensionless
d_1	Geometric constant of upper cylindrical section diameter, dimensionless
D_1	Upper cylindrical section diameter biodigester, L
d_2	Geometric constant of lower cylindrical section diameter, dimensionless
D_2	Lower cylindrical section diameter biodigester, L
D_c	Middle cylindrical section diameter biodigester, L
f_{ld}	Weight fraction of anaerobic sludge-manure in wastewater, dimensionless
g	Gravity force, Lt^{-2}
H_1	Middle cylindrical section height biodigester, L
h_1	Upper cylindrical section height biodigester, L
H_2	Lower conic section height biodigester, L
h_2	Lower cylindrical section height biodigester, L
H_3	Upper conic section height biodigester, L
h_{op}	Operation volume height biodigester, L
H_v	Hydraulic head, L
K	Consistency index, $ML^{-1} \cdot t^{n-1}$
K_1	Geometric constant of middle cylindrical section height, dimensionless
k_1	Geometric constant of upper cylindrical section height, dimensionless
K_2	Geometric constant of lower conic section height, dimensionless
k_2	Geometric constant of lower cylindrical section height, dimensionless
K_3	Geometric constant of upper conic section height, dimensionless
K_{Gp}	Constant that groups to the characteristic constant and characteristic potency exponent k_p y m, dimensionless
K_{Gr}	Constant that groups to the characteristic constant and characteristic potency exponent k_r y q, dimensionless
k_p	Constant characteristic, dimensionless
k_r	Constant characteristic, dimensionless
m	Exponent potency characteristic, dimensionless
N	Scale factor, dimensionless
n	Behavior index, dimensionless
$N_{Da\mathrm{I}}$	Damköhler I number, dimensionless
$N_{Da\mathrm{I}1}$	Damköhler I number biodigester original scale, dimensionless
$N_{Da\mathrm{I}2}$	Damköhler I number biodigester new scale, dimensionless
N_{Gi}	Geometry number, dimensionless
N_{mf}	Number of fundamental magnitudes, dimensionless
N_p	Power number, dimensionless
N_{p1}	Power number biodigester original scale, dimensionless
N_{p2}	Power number biodigester new scale, dimensionless
N_r	Recirculation number, dimensionless

N_{Re}	Reynolds number, dimensionless
N_{Re1},	Reynolds number biodigester original scale, dimensionless
N_{Re2}	Reynolds number biodigester new scale, dimensionless
N_{vp}	Number of process variables, dimensionless
P	Power consumption hydraulic recirculation of the wastewater, ML^2cdott^{-3}
p	Atmospheric pressure, $ML^{-1}t^{-2}$
P_1	Power consumption biodigester original scale, $ML^2 \cdot t^{-3}$
P_2	Power consumption biodigester new scale, $ML^2 \cdot t^{-3}$
P_{exp}	Power consumption determined experimentally, $ML^2 \cdot t^{-3}$
PM_{bg}	Biogas molecular weight, MM^{-1}
q	Exponent potency characteristic, dimensionless
R	Ideal gases constant, L^4t^{-2}
R_{bio}	Biogas production volumetric rate, $ML^{-3} \cdot t^{-3}$
R_{bio1}	Biogas production volumetric rate biodigester original scale, $ML^{-3} \cdot t^{-3}$
R_{bio2}	Biogas production volumetric rate biodigester new scale, $ML^{-3} \cdot t^{-3}$
R_{bioexp}	Biogas production volumetric rate determined experimentally, $ML^{-3} \cdot t^{-3}$
R_{rc}	Recirculation rate, t^{-1}
t_1	Geometric constant recirculation pipe intern diameter, dimensionless
T_1	Recirculation pipe intern diameter biodigester, L
t_2	Geometric constant biogas output pipe intern diameter, dimensionless
T_2	Biogas output pipe intern diameter biodigester, L
T_{op}	Temperature operation biodigester,
V_{bg}	Biogas accumulated volume, L^3
V_{biod}	Nominal volume biodigester, L^3
v_f	Wastewater recirculation velocity, $L \cdot t^{-1}$
v_{f1}	Wastewater recirculation velocity original scale biodigester, $L \cdot t^{-1}$
v_{f2}	Wastewater recirculation velocity new scale biodigester, $L \cdot t^{-1}$
V_g	Wastewater volumetric rate, L^3t^{-1}
V_{op}	Operation volume biodigester, L^3

Greek symbols

α	Parameter that groups to n and m, dimensionless
β	Parameter that groups to n and q, dimensionless
ρ	Water density, ML^{-3}
η_∞	Viscosity infinite shear, $ML^{-1} \cdot t^{-1}$
η_0	Viscosity zero shear, $ML^{-1} \cdot t^{-1}$
η_{ap}	Wastewater apparent viscosity, $ML^{-1} \cdot t^{-1}$
ρ_{ap}	Wastewater apparent density, $M \cdot L^{-3}$
ρ_{ld}	Density of the mixed of anaerobic sludge and manure, ML^{-3}
$\dot{\gamma}$	Shear velocity, t^{-1}
ϕ_p	Total suspended solids concentration, ML^{-3}
Π	Number of dimensionless pi groups, dimensionless

Note: In the nomenclature of the variables with dimensions, these were defined with the fundamental magnitudes L=longitude, M=mass and t= time.

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