Vol. 21, No. 1(2022) Proc2636
Revista Mexicana de Ingeniería Química

# A new algorithm for evaluation of kinetic parameters $R_\infty$ and $k$, in coal flotation process

# Un nuevo algoritmo para la evaluación de los parámetros cinéticos $R_\infty$ y $k$, en el proceso de flotación de carbón

E.A. González-Barraza[1], D. Martínez-Carrillo[2], H.A. Moreno-Casillas[3]*, C.M. López-Badillo[1]

[1] *Facultad de Ciencias Químicas-UA de C, Blvd. Venustiano Carranza 935, República, 25280 Saltillo; Coahuila.*
[2] *Centro de Investigación en Geociencias Aplicadas -UA de C, 5 de febrero 303-A, Independencia, 26830 Nueva Rosita; Coahuila.*
[3] *TecNM - IT la Laguna, Blvd Revolución y Av. Tecnológico de la Laguna S/N, Col. Centro, Torreón; Coah, Mexico, 27000.*

Received: November 7, 2021; Accepted: February 24, 2022

## Abstract

Floating models have been a topic of investigation for more than 90 years, which has allowed their introduction in many industrial processes. In this paper is presented, an algorithm to determine the kinetic parameters for the coal flotation process in a fast and practical way, using a spreadsheet. A $2^2$ factorial design was run with three replicates. The selected factors were: concentration of methyl isobutyl carbinol as foaming agent, and diesel as a collector. For both foaming agent and collector, low and high levels were 25 g/ton, and 100 g/ton respectively. To verify the model's curvature, an intermediate point was included. The statistical analysis of the fuel recovery and kinetic flotation is presented. Three kinetic models were used for the evaluation of the proposed algorithm, García-Zuñiga's algorithm, and both of the algorithms proposed by Klimpel. With the analysis of variance, it is concluded that both factors influence significantly the efficiency of coal flotation. However, the factor that has the most significant effect is the foaming agent. It is also concluded that the proposed algorithm is functional, presenting correlation coefficients higher than 0.9948. The utilized kinetic models present similar average $R^2$ values, for the first order $R^2$=0.9984, and for the second-order $R^2$=0.9975.

*Keywords*: flotation, kinetics, kinetic models, coal.

## Resumen

Los modelos cinéticos de flotación han sido investigados por más de 90 años y se usan en muchos procesos industriales. Para determinar la recuperación máxima ($R_\infty$) y la constante cinética de flotación ($k$), se requiere de conocimiento de la flotación, y del uso de software. Costoso para las empresas y complicado para estudiantes que inician. En este artículo se presenta un algoritmo para determinar los parámetros de forma rápida y práctica, con una hoja de cálculo. Para evaluar el método propuesto, se hicieron pruebas de flotación de carbón en una celda de laboratorio. Utilizando un diseño factorial $2^2$ y un punto intermedio, utilizando MIBC y diesel, como espumante y colector, respectivamente. Para ambos agentes el nivel alto fue de 100 g/ton y el nivel bajo de 25 g/ton. Se utilizaron 3 modelos para la evaluación del algoritmo, dos de primer orden y uno de segundo. Se presenta un análisis de la recuperación de combustible y de la cinética de flotación. Se concluye que el algoritmo propuesto es funcional, presentando coeficientes de correlación mayores a 0.9948. Se determina que la recuperación de combustible es proporcional a la concentración de reactivos y se demuestra que el espumante actúa como surfactante en el carbón. De los modelos cinéticos utilizados, los de primer orden presentan un mejor ajuste ($R^2$=0.9986) que el de segundo orden ($R^2$=0.9977).

*Palabras clave*: flotación, cinética, modelos cinéticos, carbón.

---

* *Corresponding author. E-mail*: `hamorenoc@lalaguna.tecnm.mx`
https://doi.org/10.24275/rmiq/Proc2636
*ISSN*:1665-2738, *issn-e*: 2395-8472

## 1 Introduction

Coal can be defined as a sedimentary organic rock. On a global level, carbon supplies 26% of the energetic necessities, it is capable of generating more than 41% of the electricity used by the world's population, it is utilized in the production of more than 68% of the steel, it is also a notable fuel, and plays a fundamental role in the concrete industry. For this reasons, coal demand grows each day, being produced in more than 40 countries and imported by more than 70 (Secretaría de Minería, 2020).

Mexico is known for being a mining country where its main coal deposits are located in the states of Oaxaca, Sonora, and Coahuila (Corona Esquivel *et al*, 2006), in the last one, it is located the Carboniferous Region, which has the largest coal reserve in the country, occupying the first place in the national coal production contributing in 97.46% of it (Servicio Geológico Mexicano, 2020).

Coal's quality is conditioned by the ash content, whether it is used for metallurgical purposes, thermal or even for grapheme production, hence, coal has to be put through a cleansing process before its commercialization, using the flotation process for concentration of particles finer than 0.149 mm. The ashes are composed of two parts: an inorganic (made by no crystalline solids, crystalline, and organic fluids), and an organic (made among others by sulfurs, oxides, hydroxides, silicates, sulfates, carbonates, and phosphates) (Huggins, 2002; Speight, 2005; Peña Urieta, 2011).

Coal flotation may be defined as a physicochemical process for its concentration creating the necessary conditions for the adherence of coal particles to the air bubbles. Its objective is selective separation based on the hydrophobic and hydrophilic properties of coal and/or involved materials in the process, allowing the obtainment of recovery with higher coal purity levels. Generally, the parameters that guide flotation are: the concentrations and type of collector and frother agent, the pulp's pH, speed of the air's flow, the velocity of agitation, and/or collection time. Likewise, the parameters of the coal and the equipment used need to be taken into account (Bravo-Galvez, 2004; Aldrich *et al*, 2010; Pinto-Caro, 2011; Shean *et al*, 2011; Cao *et al*, 2013; Liu *et al*, 2013; Cao *et al*, 2021; Zhu *et al*, 2021). The efficiency from which a flotation is carried out, is mainly determined for fuel recovery in percent by weight.

The kinetics of flotation has relevance because it is a study made before the coal processing or cleansing, to establish the initial operating parameters of the washing plant. A kinetic flotation model describes the evolution of valuable mineral recovery as a time function, as well as predicting the recovery at different periods. Some diverse kinetic models are applied to the flotation process, being the ones of first order, the most suitable due to their high trust level, both in batch and continuous flotation (Saleh, 2010; Gharai, 2016; Lucay *et al*, 2020; Ni *et al*, 2022).

There is a great variety of kinetic models to characterize the flotation process (Bu *et al*, 2017; Zhu *et al*, 2020). García Zuñiga, is considered the first person to create a model of the kinetics of flotation (García Zuñiga, 1935;), this model follows the mathematical equation (1).

$$\frac{dC}{dt} = -kC^n \tag{1}$$

Where $C$ is the valuable mineral concentration, $t$ is the flotation time in sec, $k$ is the flotation kinetics' constant in $\text{sec}^{-1}$, and $n$ is the reaction order.

The García-Zúñiga model, Eq. (2), However, the Klimpel model, Eq. (3).

Three models that characterize the mineral recovery in a flotation process, and adequate for coal are compared in this study, they are: García Zuñiga's first order model Eq. (2), which is the most used for carbon flotation since it considers that all particles have the same probability of floatation, Klimpel's first-order kinetic model Eq (3), which presents a lightly bigger precision for it applies a rectangular distribution to its equation, and a second-order model, also by klimpel, Eq (4) (Albijanick *et al*, 2015; Ni *et al*, 2016; Bharamia *et al*, 2019).

$$R_c = R_\infty(1 - e^{-kt}) \tag{2}$$

$$R_c = R_\infty\left[1 - \frac{1}{kt}\left(1 - e^{-kt}\right)\right] \tag{3}$$

$$R_c = R_\infty\left[1 - \frac{1}{kt}\ln(1 + kt)\right] \tag{4}$$

Where $R_c$ is the recovery of the component (%) at time $t$, $R_\infty$ is the maximum recovery (%) of the component that can be obtained from the material of interest according to the established conditions, $k$ is the flotation rate constant ($\text{s}^{-1}$) and $t$ is the accumulated flotation time (s). For the estimate of these variables, particular programs have been developed with programming software as MATLAB to complex fuzzy logic programs (Abkhoshk, *et al*, 2010), all of which look for the highest correlation coefficient ($R^2$). To find the kinetic parameters such as the maximum recovery ($R_\infty$) and the flotation rate constant ($k$), academic knowledge in the flotation process is required, and even the usage of the programming software, which could be costly for business and complicated for college students that are introduced to the basic concepts of the flotation process. This study describes a practical and simple algorithm to determine the parameters of the flotation kinetics, using the 3 kinetic models that describe the evolution of the flotation's speed. This algorithm can be very useful since only a Microsoft Excel spreadsheet is utilized. The use of specialized software is not needed nor additional tools to obtain an excellent approximation to optimal parameter values.

## 2 Materials and methods

The coal used in this investigation is from the carboniferous region, from the sub-basin Saltillito-Lampacito, located inside Aura's locality, in the state of Coahuila; Mexico. Collection of the sample was done based on the ASTM D 2234/D2234M-19 method. Accordingly, to the ASTM D2013/D2013M-20 method, it was dried at a temperature of 107 °C ± 3 °C for of 20 min and homogenized. The sample was divided into 2 sub-samples destined for the coal analysis and flotation tests, consequently, its size was reduced to particles finer than 0.250 mm, and 0.149 mm respectively.

The performed coal analyses were: moisture, ash, volatile material, sulfur, calorific power, free swelling index (FSI), density, and fixed carbon analysis, effectuated according to the corresponding ASTM methods: D720/D720M-15e1, D2234/D2234M-19, D2013/D2013M-20, ASTM D3173 /D3173M-17a, ASTM D3174-12, ASTM D3175-20, D4239-18e1, D5865/D5865M-19, and using the equipment listed in Table 1.

The results obtained from the characterization of the coal sample are shown in Table 2. According to the classification ASTM D388-19$^a$, the sample fits in the range of Sub-bituminous type A carbon.

The flotation process was executed according to the established methodology and operation conditions: The flotation pulp (10% w/w in solid content) is poured into a Denver D-12 cell and is set to a velocity of 1200 RPM in three time lapses (5 min, 2 min, and 20 sec). The first lapse (5 minutes) is destined for mixing the mineral coal in water to produce the mineral pulp, in the second lapse (2 min) the collector is homogeneously distributed in the pulp, and in the last time-lapse the foaming agent is added, and stirred for 20 seconds. After this, the air injection is started. When the foam starts to develop in the cell, it is collected, and isolated in separate containers at 30, 60, 90, 120, and 300 seconds to calculate the recovery.

Once that the tests are done, the concentrated and the sunken are filtered, then dried in an oven for a period of 10 to 15 minutes, at a temperature of 107 ± 3 °C, after drying, the weight is registered, and the ash analyses are performed. Three replicates of all of the test were performed and their average reported. All of the used reactants were of analytical grade.

Table 1. Equipment and methods for coal analysis.

| Analysis | ASTM method | Equipment |
|---|---|---|
| Humidity | D3173/D3173M-17$^a$ [26] | Oven Quincy lab |
| Ash | D3174-12 [27] | Oven Thermo Scientific |
| Volatile matter | D3175-20 [28] | Oven Thermo Scientific |
| Sulfur | D4239-18e1 [29] | Sulfur analyzer LECO SC-144DR |
| Calorific value | D5865/D5865M-19 [30] | Calorimeter LECO AC600 |
| FSI | D720/D720M-15e1 [31] | Electric oven Preiser |

Table 2. Characterization of coal sample.

| Moisture (%) | Ash (%) | Sulphur (%) | Calorific value (kcal/kg) | Volatile matter | F.S.I. | Density (g/cm$^3$) | Fixed carbon (%) |
|---|---|---|---|---|---|---|---|
| 2.755 | 3 7.74 | 0.815 | 5058.1 | 18.88 | 6.0 | 1.559 | 43.38 |

Table 3. $2^2$ experimental design.

| Effect | Factor | | g/ton | | $R_c$ | | |
|---|---|---|---|---|---|---|---|
| | A | B | A | B | | | |
| (1) | -1 | -1 | 25 | 25 | 27.76 | 25.72 | 29.29 |
| a | +1 | -1 | 100 | 25 | 43.03 | 45.02 | 41.01 |
| b | -1 | +1 | 25 | 100 | 38 | 38.5 | 37.12 |
| ab | +1 | +1 | 100 | 100 | 55.82 | 56.58 | 54.96 |

From the results, the fuel recovery is calculated, using equation (5).

$$R_c = \frac{w_c(100 - A_c)}{w_f(100 - A_f)} \times 100 \qquad (5)$$

Where $R_c$ is the fuel recovery (%), $w_c$ is the weight of the mineral in the concentrate, $w_f$ is the weight of the used feeding mineral in the flotation, $Ac$ is the ash from the mineral in the concentrate, and Af is the ash in the feeding mineral (Lopez *et al*l, 2016).

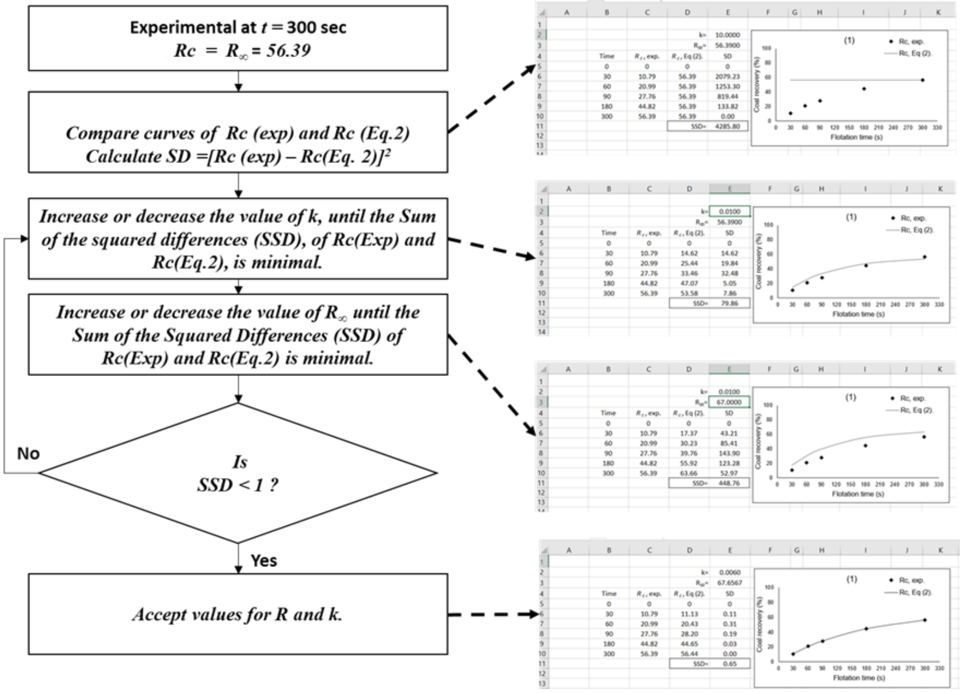
## 2.1 *Experimental design*

A $2^2$ experimental design was chosen, the considered factors were concentration of foaming agent (MIBC, [A]), and concentration of collector (diesel, [B]). Samples were obtained at 90 seconds of residence time, and three replicas were run. Factors, levels, experimental conditions, and results are shown in Table 3.

Additionally, for modeling the behavior of the coal recoveries, an intermediate (Int) run was added, the operating values for the factors were 50 g/ton of [A] and 50 g/ton of [B].

## 2.2 *The proposed algorithm for the determination of the flotation kinetics (k), and recovery at equilibrium ($R_\infty$) constants*

Figure 1, shows the flow diagram for the proposed algorithm for estimating the values for k y $R_\infty$.



k= 10.0000
$R_\infty$= 56.3900

| Time | $R_c$, exp. | $R_c$, Eq (2). | SD |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 30 | 10.79 | 56.39 | 2079.23 |
| 60 | 20.99 | 56.39 | 1253.30 |
| 90 | 27.76 | 56.39 | 819.44 |
| 180 | 44.82 | 56.39 | 133.82 |
| 300 | 56.39 | 56.39 | 0.00 |
| | | SSD= | 4285.80 |

k= 0.0100
$R_\infty$= 56.3900

| Time | $R_c$, exp. | $R_c$, Eq (2). | SD |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 30 | 10.79 | 14.62 | 14.62 |
| 60 | 20.99 | 25.44 | 19.84 |
| 90 | 27.76 | 33.46 | 32.48 |
| 180 | 44.82 | 47.07 | 5.05 |
| 300 | 56.39 | 53.58 | 7.86 |
| | | SSD= | 79.86 |

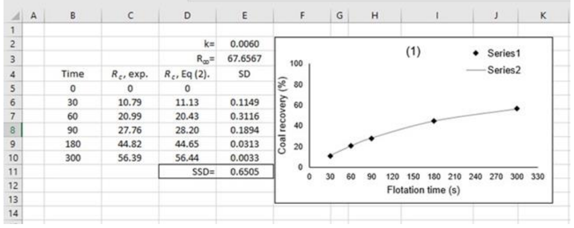
k= 0.0100
$R_\infty$= 67.0000

| Time | $R_c$, exp. | $R_c$, Eq (2). | SD |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 30 | 10.79 | 17.37 | 43.21 |
| 60 | 20.99 | 30.23 | 85.41 |
| 90 | 27.76 | 39.76 | 143.90 |
| 180 | 44.82 | 55.92 | 123.28 |
| 300 | 56.39 | 63.66 | 52.97 |
| | | SSD= | 448.76 |

k= 0.0060
$R_\infty$= 67.6567

| Time | $R_c$, exp. | $R_c$, Eq (2). | SD |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 30 | 10.79 | 11.13 | 0.11 |
| 60 | 20.99 | 20.43 | 0.31 |
| 90 | 27.76 | 28.20 | 0.19 |
| 180 | 44.82 | 44.65 | 0.03 |
| 300 | 56.39 | 56.44 | 0.00 |
| | | SSD= | 0.65 |

Figure 1. Flow diagram for the proposed algorithm.



| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | k= | 0.0060 |
| 3 | | | | $R_\infty$= | 67.6567 |
| 4 | | Time | $R_c$, exp. | $R_c$, Eq (2). | SD |
| 5 | | 0 | 0 | 0 | |
| 6 | | 30 | 10.79 | 11.13 | 0.1149 |
| 7 | | 60 | 20.99 | 20.43 | 0.3116 |
| 8 | | 90 | 27.76 | 28.20 | 0.1894 |
| 9 | | 180 | 44.82 | 44.65 | 0.0313 |
| 10 | | 300 | 56.39 | 56.44 | 0.0033 |
| 11 | | | | SSD= | 0.6505 |

Figure 2. Parameter estimation for equation (2), using the proposed algorithm.

### 2.2.1 *Kinetics parameters estimation (k y $R_\infty$)*

An Excel (Microsoft) data sheet is used for this algorithm. Figure 2, shows data for kinetics parameters estimation only for equation (2). Two cells are specified for the values for $k$ (E2) and $R_\infty$ (E3), one column for residence time, another one for the experimental data ($R_c$), and for each of the models Eq (2), (3) and (4). The obtained results are compared to those from the least squares method. SD is the squared difference between experimental $R_c$, and the model $R_c$. The same procedure is followed for equations (3) and (4).

# 3 Results and discussion

## 3.1 *Coal recovery*

Figure 3 shows the graphs for coal recovery versus residence time of each experimental condition. At 30 sec, coal recovery values vary from 10.79 % for experimental condition (1), up to a 22.42 % for experimental condition (ab). (a), and (b) are very close and slightly above the average. Coal recovery increases quasi-proportional up to 90 sec in all cases, which indicates that mineral particles are activated. Curves generated for each experimental condition have the same tendency, in ascending order (1), (b), (Int), (a) and (ab). Compared to the results at 30 sec, at the end of the test (300 sec), the differences between fuel recoveries are bigger, 36.38% for experimental condition (1), and 84.45% for experimental condition (ab).

From the pattern of behavior for fuel recovery ($R_c$) during the test, it can be concluded that the foaming agent (MIBC), has a significant effect on the recovery of the coal particles, and it might be because it is also acting as a surfactant [32].

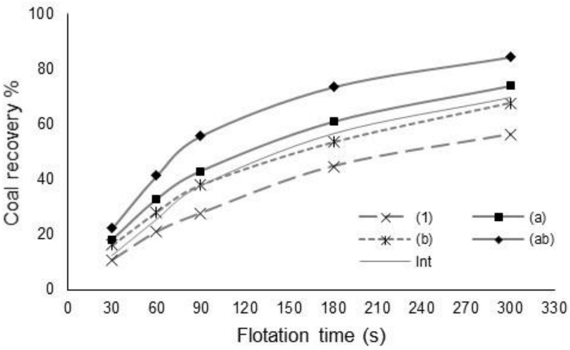


Figure 3. Coal recovery for the performed tests.

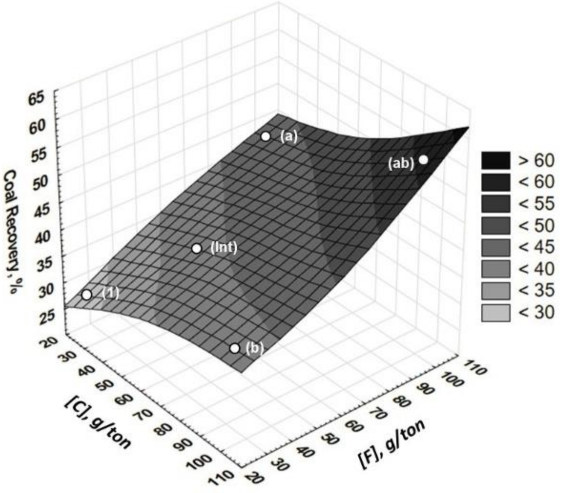
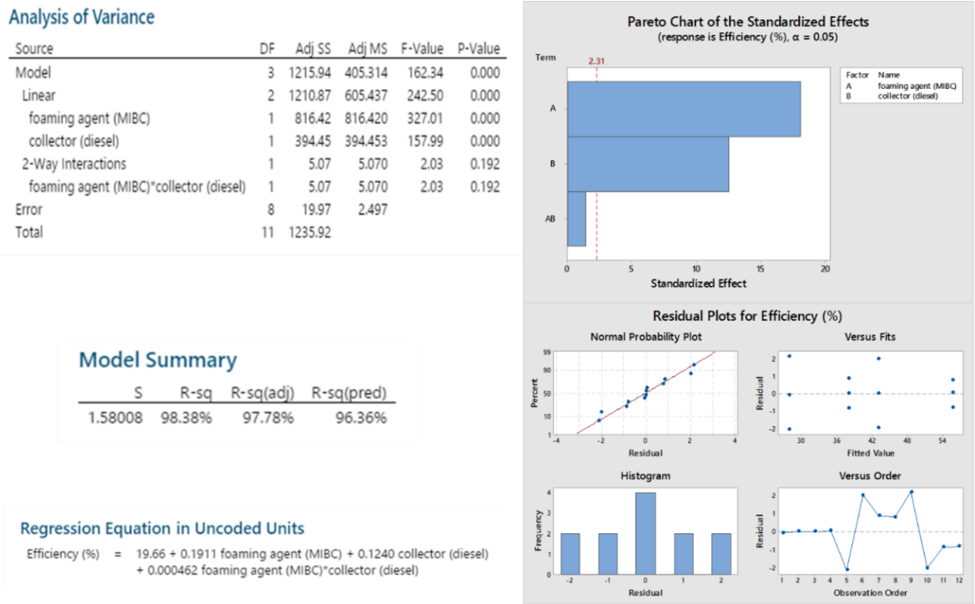


Figure 4. Coal recovery surface curve.

In the fuel recovery graph (figure 3), a change of slope occurs at 90 sec, taking this breaking point and the corresponding data from the experimental design, a surface graph was obtained in order to observe the behavior of coal recovery (figure 4). It can be seen that coal recovery increases with concentration of reactants, and that the foaming agent (A), has a bigger effect on coal recovery than the collector (B).

The Analysis of Variance (ANOVA), using MINITAB software, was also run with the data corresponding to 90 sec of residence time (figure 5).

Results confirm that the factor that has the more significant effect on coal recovery is the foaming agent (MIBC), followed for the collector (diesel); while the interaction has no significant effect. The Residuals Analysis indicates that the ANOVA assumptions are fulfilled, and that the model is adequate. The positive signs of the coefficients indicate that increasing the amount of reactants, coal recovery efficiency increases.



**Analysis of Variance**

| Source | DF | Adj SS | Adj MS | F-Value | P-Value |
|---|---|---|---|---|---|
| Model | 3 | 1215.94 | 405.314 | 162.34 | 0.000 |
| Linear | 2 | 1210.87 | 605.437 | 242.50 | 0.000 |
| foaming agent (MIBC) | 1 | 816.42 | 816.420 | 327.01 | 0.000 |
| collector (diesel) | 1 | 394.45 | 394.453 | 157.99 | 0.000 |
| 2-Way Interactions | 1 | 5.07 | 5.070 | 2.03 | 0.192 |
| foaming agent (MIBC)*collector (diesel) | 1 | 5.07 | 5.070 | 2.03 | 0.192 |
| Error | 8 | 19.97 | 2.497 | | |
| Total | 11 | 1235.92 | | | |

**Model Summary**

| S | R-sq | R-sq(adj) | R-sq(pred) |
|---|---|---|---|
| 1.58008 | 98.38% | 97.78% | 96.36% |

**Regression Equation in Uncoded Units**

Efficiency (%) = 19.66 + 0.1911 foaming agent (MIBC) + 0.1240 collector (diesel)
+ 0.000462 foaming agent (MIBC)*collector (diesel)

Figure 5. ANOVA for coal recovery.

The bigger effect of the MIBC can be attributed to the surfactant property of this agent on carbon.

## 3.2 *Flotation kinetics*

Using the proposed algorithm, for kinetic parameters estimation for each of the models the results were quite acceptable. Figure 6, shows an acceptable fit of the kinetics models to the experimental data for all of the performed tests.

Table 5, shows the statistics obtained for the kinetics parameters estimation of the considered models. The dispersion of data or Sum of Squares for Error (SSE) is also included. $R^2$ values close to 1 indicate a high correlation for each of the models, with better fit for first order model, equations 2 and 3.

In order to get the best fit of kinetics models to experimental data, and for comparison, the least squares method was used, the correspondent graphs are shown in figure 7. Least squares models are best fitted to experimental data.

In table 5, the statistical data for estimation of kinetics parameters using the least square method are presented. As in table 4, there is a good correlation between modeled and experimental data, even more they are similar. However, the SSE in table 5, is smaller in all cases, with a bigger difference for the first order kinetic model, equation (3).

Comparing the three models for this particular case it can be observed that dispersion of data for the first order models, equations 2 and 3 respectively, are really close (5.24 and 5.09), as it is for their correlation coefficients (0.9986 y 0.9987). While the second order kinetic model, equation 4, shows a bigger dispersion (9.61), and a slightly lower correlation coefficient of 0.9977.

Table 4. $k$ and $R_\infty$ for the kinetics models using the proposed algorithm.

| Test | Eq. (2) | | | | Eq. (3) | | | | Eq. (4) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R_\infty$ | $k$ | SSE | $R^2$ | $R_\infty$ | $k$ | SSE | $R^2$ | $R_\infty$ | $k$ | SSE | $R^2$ |
| -1 | 69.3 | 0.0057 | 1.1246 | 0.9996 | 80 | 0.011 | 3.6531 | 0.9993 | 124.5 | 0.0068 | 3.8386 | 0.9987 |
| (a) | 79.5 | 0.0089 | 6.946 | 0.999 | 96 | 0.0148 | 2.845 | 0.9996 | 131.2 | 0.0119 | 4.0058 | 0.999 |
| (b) | 76 | 0.0075 | 9.1524 | 0.9978 | 89.5 | 0.0136 | 6.5984 | 0.9986 | 121 | 0.011 | 1.8909 | 0.9994 |
| (ab) | 87.5 | 0.0107 | 7.079 | 0.9987 | 103.5 | 0.0176 | 23.9904 | 0.9973 | 134.5 | 0.0165 | 27.1734 | 0.9948 |
| (Int) | 85.5 | 0.0057 | 16.1919 | 0.9965 | 99 | 0.0104 | 18.4865 | 0.9973 | 132 | 0.0097 | 22.8198 | 0.9956 |
| Average | 79.56 | 0.0077 | 8.0988 | 0.9983 | 93.6 | 0.0135 | 11.1147 | 0.9984 | 128.64 | 0.0112 | 11.9457 | 0.9975 |

Table 5. $k$ and $R_\infty$ for the floatation kinetic models using the least square method.

| Test | Eq. (2) | | | | Eq. (3) | | | | Eq. (4) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R_\infty$ | $k$ | SSE | $R^2$ | $R_\infty$ | $k$ | SSE | $R^2$ | $R_\infty$ | $k$ | SSE | $R^2$ |
| -1 | 67.66 | 0.006 | 0.6505 | 0.9997 | 82.79 | 0.01 | 0.8333 | 0.9996 | 120.25 | 0.0074 | 2.205 | 0.9991 |
| (a) | 79.15 | 0.0086 | 3.315 | 0.9991 | 93.99 | 0.0152 | 1.3043 | 0.9996 | 125.94 | 0.0129 | 2.4417 | 0.9994 |
| (b) | 73.74 | 0.0078 | 7.2178 | 0.9977 | 88.12 | 0.0136 | 4.3931 | 0.9986 | 120.14 | 0.0112 | 1.8044 | 0.9994 |
| (ab) | 87.68 | 0.0106 | 7.0554 | 0.9986 | 102.59 | 0.0192 | 9.2679 | 0.9983 | 132.41 | 0.0174 | 25.4757 | 0.9951 |
| (Int) | 81.01 | 0.0064 | 7.7305 | 0.998 | 98.9 | 0.0107 | 9.37 | 0.9975 | 142.87 | 0.008 | 15.6769 | 0.9957 |
| Average | 77.85 | 0.0079 | 5.1938 | 0.9986 | 93.28 | 0.0137 | 5.0337 | 0.9987 | 128.32 | 0.0114 | 9.5207 | 0.9977 |

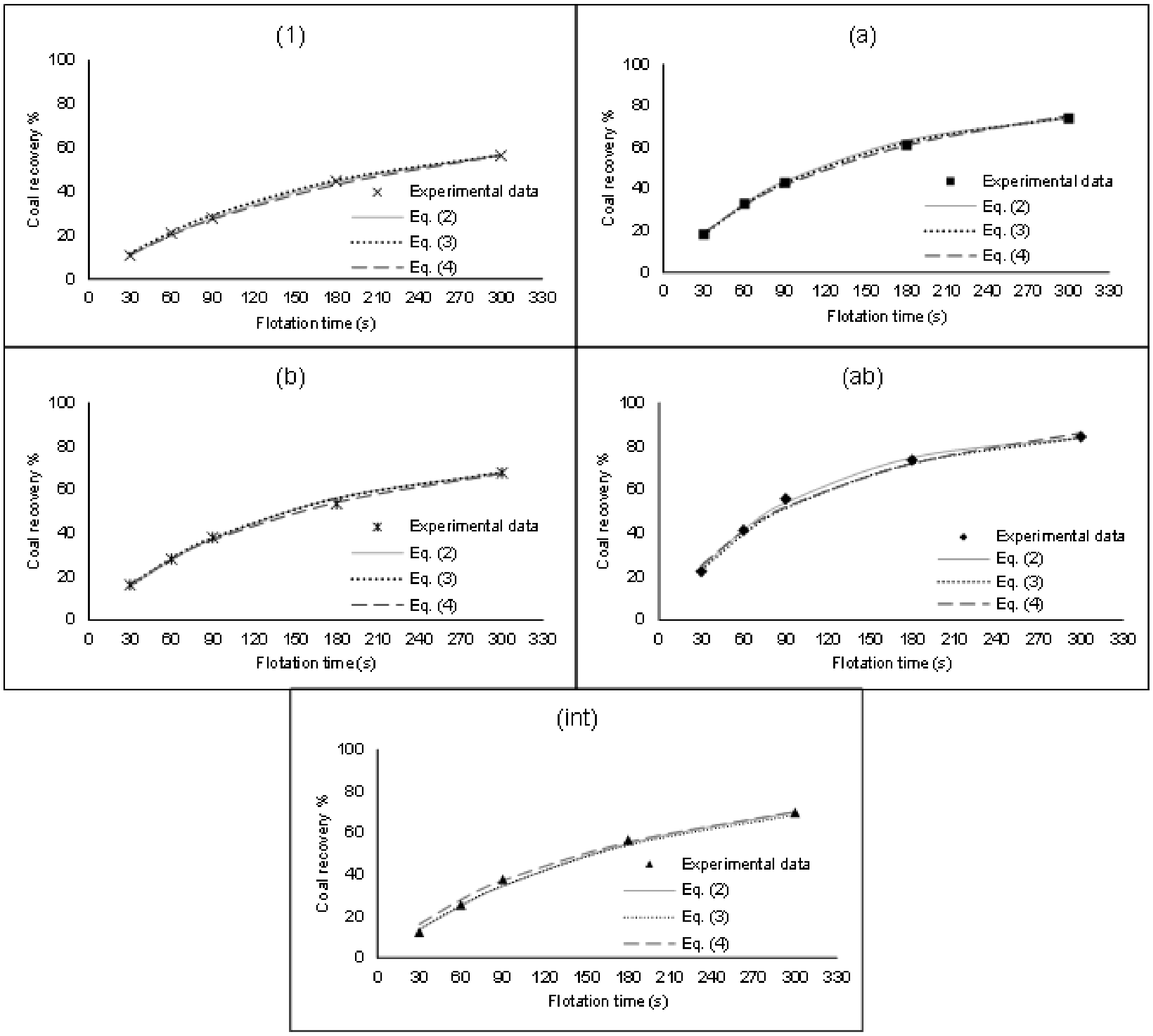


Figure 6. Graphs for the kinetics models using the proposed algorithm.

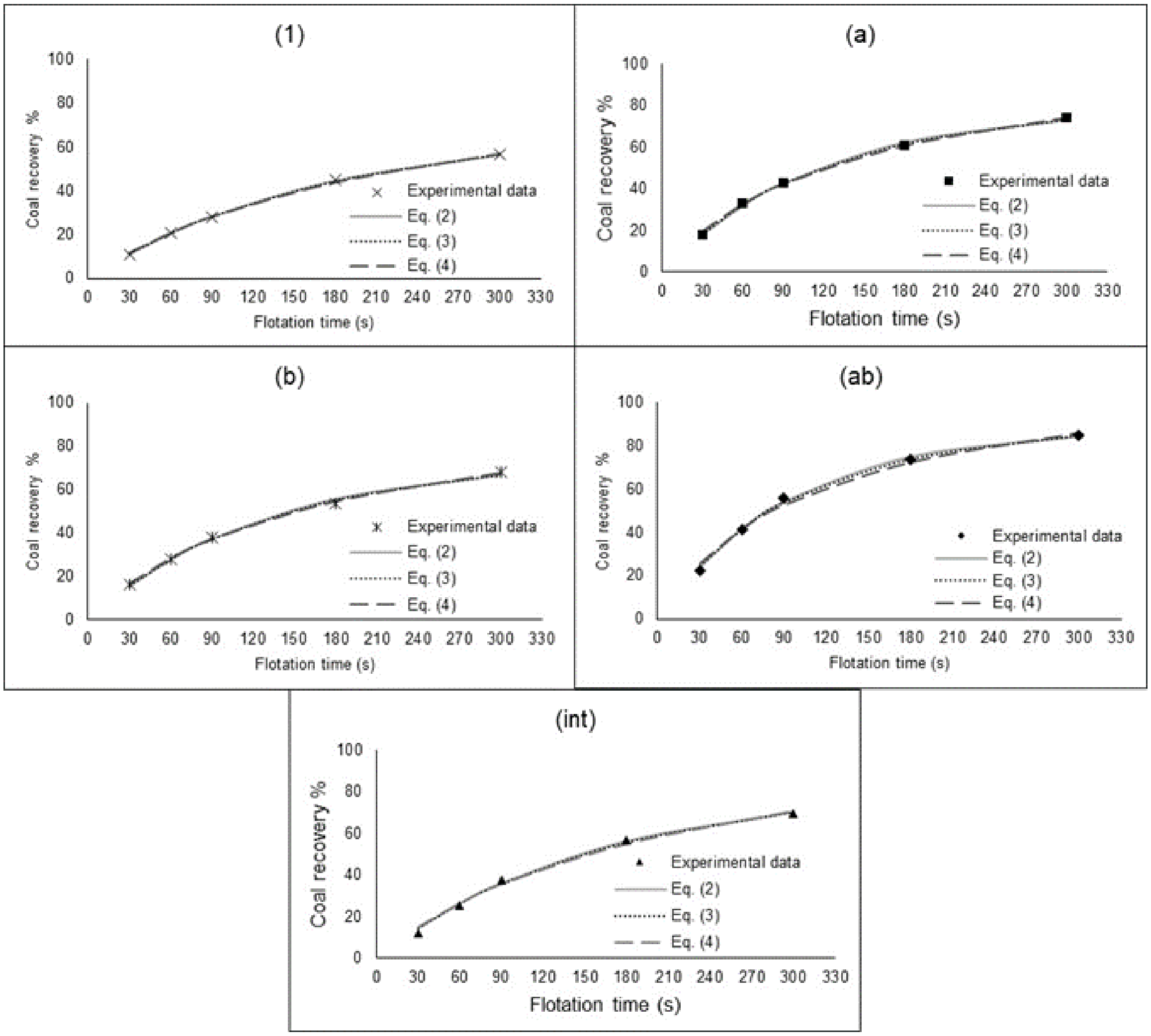


Figure 7. Kinetic models using the least squares method.

## Conclusions

The proposed algorithm, had an excellent adjustment to the experimental data for all three kinetic models, getting a maximum correlation coefficient ($R^2$) of 0.9996, and a minimum of 0.9948. Besides, it is easy to use, and Excel is available in most computers.

The performed flotation tests showed that both factors; concentration of MIBC and diesel have a significant effect on the fuel recovery. MIBC has the bigger effect, and also acts as a surfactant in the carbon flotation process as mentioned in the related literature. Interaction of factors has no significant effect on the fuel recovery.

The best experimental results were obtained with the experimental condition (ab), reaching a value of 84.45%. The lower value for fuel recovery corresponded to the experimental condition with the low level of factors (1), and it was of 56.39 %.

According to the statistic results presented, the first order kinetics models have a better adjustment to experimental data than the second order kinetic model.

### *Acknowledgments*

To the Autonomous University of Coahuila (UAC) for the support for this project, especially to the Research Center of Applied Geosciences for sample analysis, to the Superior Engineering School for let us use their flotation equipment, and to the TecNM, IT la Laguna.

## References

Abkhoshk, E. , Kor, M. and Rezai, B. (2010). A study on the effect of particle size on coal flotation kinetics using fuzzy logic. *Expert System with Applications 37(7)*, 5201-5207. DOI: 10.1016/j.eswa.2009.12.071

Ai, G., Yang, X. and Li, X. (2017). Flotation characteristics and flotation kinetics of fine wolframite. *Powder Technology 305*, 377-381. https://doi.org/10.1016/j.powtec.2016.09.068

Albijanic, B., Subasinghe, N. and Park, C.H. (2015). Flotation kinetic models for fixed and variable pulp chemical conditions. *Minerals Engineering 78*, 66-68. https://doi.org/10.1016/j.mineng.2015.04.010

Aldrich, C., Marais, C., Shean, B. and Cilliers, J. (2010). Online monitoring and control of froth flotation systems with machine vision: a review. *International Journal of Mineral Process 96(1-4)*, 1-13. http://dx.doi.org/10.1016/j.minpro.2010.04.005

ASTM D388-19a Standard Classification of Coals by Rank, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D720/D720M-15e1 Standard Test Method for Free-Swelling Index of Coal, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D2234/D2234M-19 Standard Practice for Collection of a Gross Sample of Coal, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D2013/D2013M-20 Standard Method of Preparing Coal Samples for Analysis, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D3173 /D3173M-17a Standard Test Method for Moisture in the Analysis Sample of Coal and Coke, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D3174-12 Standard Test Method for Ash in the Analysis Sample of Coal and Coke from Coal, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D3175-20 Standard Test Method for Volatile Matter in the Analysis Sample of Coal and Coke, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D4239-18e1 Standard Test Method for Sulfur in the Analysis Sample of Coal and Coke Using High-Temperature Tube Furnace Combustion, ASTM International, West Conshohocken, PA, www.astm.org

ASTM D5865/D5865M-19 Standard Test Method for Gross Calorific Value of Coal and Coke, ASTM International, West Conshohocken, PA, www.astm.org

Bahramia, A., Kazemib, F. and Ghorbanic, Y. (2019). Effect of different reagent regime on the kinetic model and recovery in gilsonite flotation. *Journal of Materials Research and Technology 8(5)*, 4498-4509. https://doi.org/10.1016/j.jmrt.2019.07.063

Bravo-Gálvez, A.C., (2004). Planta concentradora manual de flotación. Casapalca. Available at: https://www.academia.edu/8855584/Planta_Concentradora. Accessed October 29, 2021.

Bu, X., Xie, G., Peng, Y., Ge, L. and Ni, C. (2017). Kinetics of Flotation. Order of process, rate constant distribution and ultimate recovery. *Physicochemical Problems of Mineral Processing 53(1)*, 342-365. https://doi.org/10.5277/ppmp170128

Cao, B., Xie, Y., Gui, W., Wei, L., and Yang, C. (2013). Integrated prediction model of bauxite concentrate grade based on distributed machine vision. *Minerals Engineering 53*, 31-38. https://doi.org/10.1016/j.mineng.2013.07.003

Cao, S., Yin, W., Yang, B., Zhu, Z., Sun, H., Sheng, Q., and Che, K. (2021). Insights into the influence of temperature on the adsorption behavior of sodium oleate and its response to flotation of quartz. *International Journal of Mining Science and Technology*. https://doi.org/10.1016/j.ijmst.2021.12.006

Corona-Esquivel, R., Tritlla, J., Muñoz, M., Piedad-Sanchez, N., and Villafranca, I. (2006). Geología, estructura y composición de los principales yacimientos de carbón mineral en México. *Boletín de la Sociedad Geológica Mexicana 58(1)*, 141-160. http://boletinsgm.igeolcu.unam.mx/bsgm/vols/epoca04/5801/(5)Corona.pdf

Dowling, E.C., Klimpel, R.R. and Aplan F.F. (1985). Model Discrimination in the flotation of a porphyry copper ore. *Minerals and Metallurgical Processing* 2, 87-101. https://doi.org/10.1007/BF03402602

Drzymala J. and Kowalczuk, P.B. (2018). Classification of flotation frothers. *Minerals 8(2)*, 53. https://doi.org/10.3390/min8020053

García-Zuñiga, H. (1935). La recuperación por flotación es una función exponencial del tiempo. *Boletín Minero de la Sociedad Nacional Minería Chile 47(418)*, 83-86. Available at: http://www.bibliotecanacionaldigital.gob.cl/colecciones/BND/00/RE/RE0000542_0164.pdf Accesed October 29, 2021.

Gharai, M. and Venugopal, R. (2016). Modeling of flotation process-An overview of different approaches. *Mineral Processing and Extractive Metallurgy Review 37(2)*, 120-133. DOI: 10.1080/08827508.2015.1115991

Huggins, F.E. (2002). Overview of analytical methods for inorganic constituents in coal. *International Journal of Coal Geology 50*, 169-214. https://doi.org/10.1016/S0166-5162(02)00118-0

Liu, J., Gui, W., Tang, Z., Yang, C., Zhu, J. and Li, J. (2013). Recognition of the operational statuses of reagent addition using dynamic bubble size distribution in copper flotation process. *Minerals Engineering 45*, 128-141. https://doi.org/10.1016/j.mineng.2013.02.003

López-Saucedo, F., Pecina-Treviño, E., De la Garza-Rodríguez, I., Ramos-Méndez, K., Camacho-Ortegón, L., and Equihua-Guillén, F. (2016). Efecto de soluciones de KI, NaCl, $MgCl_2$ y $Na_2SO_4$ en la distribución de tamaños de burbuja y su relación con la flotación de partículas de carbón y materia mineral. *Revista Mexicana de Ingeniería Química 15(1)*, 221-229. http://www.rmiq.org/ojs311/index.php/rmiq/article/view/1114/419

Lucay, F.A., López-Arenas, T., Sales Cruz, M., Galvez, E.D., and Cisternas, L.A. (2020). Performance profiles for benchmarking of global sensitivity analysis algorithms. Revista Mexicana de Ingeniería Química, 19(1), 423-444. https://doi.org/10.24275/rmiq/Sim547

Ni, C., Xie, G., Jin, M., Peng, Y. and Xia W. (2016). The difference in flotation kinetics of various size fractions of bituminous coal between rougher and cleaner flotation processes. *Powder Technology 292*, 201-216. https://doi.org/10.1016/j.powtec.2016.02.004

Ni, C., Zhang, Q., Jin, M., Xie, G., Peng, Y., Yu, H., and Bu, X. (2022). Effect of high-speed shear flocculation on the flotation kinetics of ultrafine microcrystalline graphite. *Powder Technology 396*, 345-353. https://doi.org/10.1016/j.powtec.2021.10.041

Peña Urueña, M.L. (2011). Caracterización de cenizas de algunos carbones colombianos in situ por retrodispersión gamma-gamma. Tesis de Maestría en Ciencias Químicas. Universidad Nacional de Colombia, Colombia. https://repositorio.unal.edu.co/bitstream/handle/unal/8629/maryluzpenauruena.2011.pdf?sequence=1&isAllowed=y

Pinto-Caro, D.A. (2011). Implementación de una columna de flotación para reducir el contenido de cenizas en carbones Magallánicos. Tesis de Ingeniero Civil Químico. Universidad de Magallanes, Chile. http://www.umag.cl/biblioteca/tesis/pinto_caro_2011.pdf

Saleh, A.M. (2010). A study on the performance of second order models and two phase models in iron ore flotation. *Physicochemical Problems of Mineral Processing 44(1)*, 215-230.

Secretaría de Minería (2020). Perfil de Mercado del Carbón. México. Available at: https://www.gob.mx/cms/uploads/attachment/file/564112/Carb_n_2020__ENE_.pdf Accessed October 29, 2021.

SGM (2020). Panorama minero del estado de Coahuila. México. Available at: http://www.sgm.gob.mx/pdfs/COAHUILA.pdf. Accessed October 29, 2021.

Shean, B. and Cilliers, J. (2011). A review of froth flotation control. *International Journal of Mineral Processing 100*, 57-71. https://doi.org/10.1016/j.minpro.2011.05.002

Speight, J. G. (2005). *Handbook of Coal Analysis*. John Wiley & Sons, USA.

Zhu, C., Li, G., Xing, Y., and Gui, X. (2021). Adhesion forces for water/oil droplet and bubble on coking coal surfaces with different roughness. *International Journal of Mining Science and Technology 31(4)*, 681-687. https://doi.org/10.1016/j.ijmst.2021.03.002

Zhu, H., Li, Y., Lartey, C., Li, W., and Qian, G.(2020). Flotation kinetics of molybdenite in common sulfate salt solutions. *Minerals Engineering 148*, 106182. https://doi.org/10.1016/j.mineng.2020.106182