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Adaptive sliding mode control of a bioreactor via nutrients variable control using a diffeomorphism

Control adaptativo por modos deslizantes de un biorreactor mediante el control de la variable de nutrientes usando un difeomorfismo

A.E. Rodriguez-Mata[1], J.A. Medrano-Hermosillo[1*], V.A. Gonzalez-Huitron[2], L. Djilali[1], P.A. Lopez-Perez[3], A. Sonck[1]

[1]Division of Graduate Studies and Research, Chihuahua Technological Institute,
[2]Division of Graduate Studies and Research, Queretaro Technological Institute, Queretaro,
[3]Apan High School, Universidad Autónoma del Estado de Hidalgo, Carretera Apan-Calpulalpan km. 8, Apan Hidalgo, C.P. 43900, Mexico.

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Abstract

Bioengineering has advanced thanks to progress in chemical engineering and control theory, leading to the development and production of innovative drugs and medical treatments. Our research introduces a new and reliable adaptive sliding mode control (ASMC) method that involves an initial mixing phase of the substrate before injecting it into the bioreactor, focusing on regulating nutrient concentrations. By using a diffeomorphism, we break down mathematical properties like bilinearity, enabling us to create a a chain-like structure system to implement nonlinear ASMC effectively. This fresh approach shows promise in enhancing the efficiency and output of fermentation processes, opening up new possibilities for controlling biomass production in the fermentation industry.

Keywords: Bioreactor, adaptive sliding modes, nutrient concentration control, bilinearity elimination.

Resumen

La bioingeniería ha avanzado gracias al progreso en la ingeniería química y la teoría de control, lo que ha llevado al desarrollo y producción de medicamentos innovadores y tratamientos médicos. Nuestra investigación introduce un nuevo y confiable método de control adaptativo por modos deslizantes (ASMC) que involucra una fase inicial de mezcla del sustrato antes de inyectarlo en el biorreactor, enfocándose en regular las concentraciones de nutrientes. Mediante el uso de un difeomorfismo, desglosamos propiedades matemáticas como la bilinealidad, lo que nos permite crear un sistema de estructura similar a una cadena para implementar el ASMC no lineal de manera efectiva. Este enfoque novedoso se muestra prometedor en mejorar la eficiencia y el rendimiento de los procesos de fermentación, abriendo nuevas posibilidades para controlar la producción de biomasa en la industria de la fermentación.

Palabras clave: Biorreactor, modos deslizantes adaptativos, control de concentración de nutrientes, eliminación de bilinealida.

*Corresponding author. E-mail: jesus.mh@chihauahua.tecnm.mx;
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1 Introduction

Bioengineering, a constantly evolving field, is prized for its interdisciplinary nature, combining aspects of biology, chemistry, electronics, physics, and engineering to drive advances that impact diverse industries (Chao *et al*., 2017; Moralez *et al*., 2010). The field of bioengineering in commercial applications of biotechnology has triggered significant changes in the processes of healthcare, agriculture and chemical production to support the growing demand. These changes go beyond small improvements; they redefine what can be achieved by transforming medical treatments, increasing the nutritional value and resilience of crops, and improving the efficiency of chemical manufacturing processes (KesikBrodacka, 2017).

In bioengineering, fermentation is a field where precise control of biological processes is crucial. Adjusting fermentation conditions, such as pH and temperature, is key to maximize productivity and optimize efficiency in industries that rely on biotechnological methods, although such variables are already widely studied and implemented in the majority self-sustaining processes (Lisci *et al*., 2021). The most important variables to control will always be the concentration of biomass, nutrients, and the different sub-products of interest (Rodríguez-Mariano et., 2015). The importance of these conditions underlines the need for control mechanisms in the design and operation of bioreactors. For example, different control methods have been suggested, such as using control to monitor product concentrations with adaptive control for milk fermentation (Ritonja {\it et al}., 2020). Specific applications include the use of a heating system to make drinks and the creation of a control set-up for electro-fermentation processes (Haidar *et al*., 2022). The intricate nature of bioprocessing in sectors such as biopharmaceuticals has highlighted the need for advanced control systems (Mitra and Murthy, 2021). Furthermore, research has delved into designing temperature controls for solid state fermentation to improve enzyme production efficiency (Sudha *et al*., 2021).

In this context, the Monod model emerges as a theoretical framework, with practical implications, that effectively balances analytical simplicity with biological accuracy, which is why this type of model continues to be used in the design of fermentation automation systems in bioreactors today. The Monod model, known for explaining how microbial growth rates are related to substrate concentration (Lopéz-Peréz *et al*., 2022a; Prasad {\it et al}., 2019; Aguilar *et al*., 2004). Its versatility and effectiveness in bioprocessing have established its reputation as a tool of choice among industry experts and practitioners. The enduring relevance of the Monod model demonstrates its ability to provide meaningful and practical insights for fine-tuning bioreactor configurations to optimize microbial growth and product development. And with the help of feedback control, it is possible to reject phenomena or perturbations not taken into account with this type of modeling (Lopéz-Peréz *et al*., 2022a).

The wide acceptance of the Monod model in industrial circles demonstrates its significant impact on bioengineering (Alvarez-Ramirez *et al*., 2019; Gorin and Pachter, 2022). It is an example of how theoretical models can go beyond discussion to bring about real improvements in bioprocess efficiency and effectiveness. This synergy between theory and application reflects the values of bioengineering, where collaborative efforts between disciplines drive innovation.

Recent advances in control theory have led to the development of robust adaptive controllers capable of handling uncertainties and disturbances in nonlinear systems. For example, a robust adaptive controller designed for bioreactor applications was introduced in de Souza *et al*. (2020). Similarly, Vásquez *et al*. (2023) proposed a sliding mode controller for non-linear systems with indeterminate parameters, capable of managing parametric uncertainties and external disturbances. These developments underscore the progress in the design of controllers that can adapt and maintain robustness, even in the inherently unstable environment of bioreactors Rodríguez-Mata *et al*. (2020).

Various research studies have explored the use of sliding mode control in bioreactors. In the work (Rodríguez-Mata {\it et al}., 2020; Lopéz-Peréz *et al*., 2022b; Aguilar *et al*., 2010; Aguilar-López {\it et al}., 2006) they propose sliding mode controllers for managing microalgae and bioreactor cultures, respectively, to address system interruptions and excess metabolism. Other, such as (Estrada and Camacho, 2021) and (Zlateva, 2020) expanded on this by applying modified sliding mode control to address pH neutralization and biogas production in bioreactor systems. Together, these studies show how sliding-mode control can effectively handle the dynamics and uncertainties present in bioreactors. But in this case, the control variable has always been proposed as the dilution rate, which is a relationship between flow and volume, for when the volume is constant, you often have the problem of negative saturation, which is a problem in real implementation. Furthermore, the use of the dilution rate as a control variable can complicate the dynamics of the system, as evidenced in Yabo *et al*. (2020). An alternative approach involves using nutrient concentration as the control variable while keeping the dilution rate constant. This method allows for more precise control and can be implemented using a control and mixing device to estimate the nutrient concentration Ekpenyong *et al.* (2021). In works such as (Selisteanu *et al.*, 2007) it has been possible to study the combination between the sliding mode controller for bioreactors, but unlike this work, this work is proposed on the development of a new control variable (nutrient concentration) and via a direct design on a complete diffeomorphism. In works like (Picó-Marco *et al.*, 2005) it has been possible to study the adaptive sliding mod control applied to a feedbach fermentation process, but it lacks a design of the complete (diffeomorphic) model.

In this work we propose a complete analysis for a new control variable (concentration) and for a continuous process. Despite these advances, there is a paucity of research that focuses on using input nutrient concentration as a control parameter to maintain a consistent dilution rate while incorporating robust control mechanisms to handle process disturbances. In this study, our aim is to fill this gap by developing a diffeomorphism-based controller. This controller disrupts theoretical properties like bilinearity to establish a chain of integrators and implements a nonlinear adaptive sliding mode control (ASMC) strategy. Inspired by electromechanical systems, this controller will be applied to bioreactor control, representing a paradigm shift in the field.

The paper is organized as follows. Section I offers an introductory overview and a review of the current state of the art. Section II introduces a dynamic and problematic model proposed for further investigation. Section III presents the primary outcomes related to the development of a versatile sliding-mode controller for a diffeomorphic biological system. Section IV includes a numerical study that simulates bioprocess control for robust biomass production, and Section V concludes with a discussion and conclusion.

2 Statement problem and mathematical fundamentals

In a continuous reactor, the standard metabolic model for microorganism culture is described below. For x_1 and x_2 are state variables representing, respectively, the dynamics of biomass and nutrients. Parameters a_1, a_2, and a_3 denote maximal growth rate, saturation constant, and metabolic stoichiometry constant, respectively. a_4 signifies the dilution rate, while $u(t)$ represents the concentration of nutrients in the reactor input control. Employing $u(t)$ as the control variable yields a typical non-linear model, contrasting with the bi-linear model obtained when using the dilution rate as the control. This paper will propose the basic use of a bioreactor model to illustrate bioprocess dynamics and the difficulties in controlling them using robust nonlinear techniques. Bioreactor models rationalize biochemical engineering, illustrating basic principles and preliminary control strategies. The robust control algorithm provides robustness, enabling its application in more complex models. The effectiveness and simplicity of these models' applications determine the efficiency of the methods in more intricate systems. The scientific literature recognizes the importance of simple models in the development of control strategies (Rathore *et al.*, 2021). They streamline and clarify a system's behavior. This optimizes bioprocessing strategies. The dynamic model is formulated as follows.

$$\begin{aligned} \frac{dx_1}{dt} &= (\mu(\cdot) - a_4)\, x_1 \\ \frac{dx_2}{dt} &= -a_3\mu(\cdot)x_1 - a_4\,(x_2 - u(t)) \\ \mu(\cdot) &= \frac{a_1 x_2}{a_2 + x_2} \end{aligned} \tag{1}$$

Upon proposing a basis change, the diffeomorphism is calculated as $z_1 = ln(x_1)$ and $\dot{z}_1 = \dot{x}_1 x_1^{-1}$. Part of the diffeomorphism used is inspired by (Celikovsky *et al.*,2015), but has been adapted for the new control variable, which is the concentration of nutrients $u(t)$. Hence, it is follows:

$$\begin{aligned} \frac{dz_1}{dt} &= \mu(\cdot) - a_4 \\ \frac{dx_2}{dt} &= -a_3\mu(\cdot)\exp(z_1) - a_4 x_2 + a_4 u(t) \\ \mu(\cdot) &= \frac{a_1 x_2}{a_2 + x_2} \end{aligned} \tag{2}$$

Where $\mu(\cdot)$ is the rate of biological growth in the bioreactor. If the following base change is proposed $z_2 = \frac{a_1 x_2}{a_2+x_2} - a_4 = \mu(\cdot) - a_4$. Hence $x_2 = -\frac{a_2(z_2+a_4)}{z_2+a_4-a_1} \rightarrow x_2 > 0\ \ \forall a_1 > z_2 + a_4$. If derived with respect to time, the following diffeomorphism is reached:

$$\frac{dx_2}{dt} = \frac{a_2 a_1}{(z_2 + a_4 - a_1)^2}\frac{dz_2}{dt} \tag{3}$$

If we substitute (3) in (2) with $f_{z_2}(t) = (z_2 + a_4 - a_1)^2, \theta_1 = \frac{a_3}{a_2 a_1}$, $\theta_2 = \frac{a_4}{a_1}$ and $\theta_3 = \frac{a_4}{a_2 a_1}$:

$$\begin{aligned} \frac{dz_1}{dt} &= z_2 \\ \frac{dz_2}{dt} &= -\theta_1\,(z_2 + a_4)\exp(z_1)\, f_{z_2}(t) \\ &\quad + \theta_2\,(z_2 + a_4)\sqrt{f_{z_2}(t)} + \theta_3 f_{z_2}(t)u(t) + \delta(t) \end{aligned} \tag{4}$$

The term $\delta(t)$ is introduced to account for uncertainties and parametric bounds, constrained as $|\delta(t)| < k_1 z_1 < k_1 \ln|x_1|$. This perturbation is related to the dynamics of the natural logarithm of total biomass. A basis change transforms the system into a second-order equation, facilitating the development of a robust

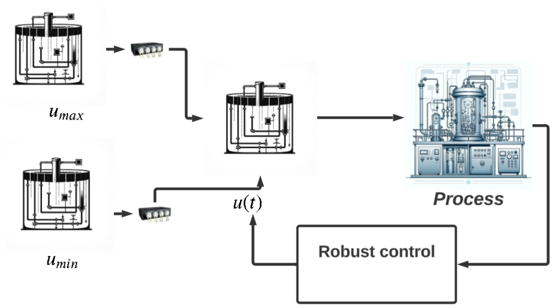

Figure 1. Schematic diagram of the control.

sliding-mode control law based on electro-mechanical design. Consequently, $z_1 = y$, $\dot{y} = \dot{z}_1 = z_2$, and $\ddot{y} = \dot{z}_2$.

$$\ddot{y} = -\theta_1 (\dot{y} + a_4) \exp(y) f_{z_2}(t) + \theta_2 (\dot{y} + a_4) \sqrt{f_{z_2}(t)} + \theta_3 f_{z_2}(t) u(t) + \delta(t) \tag{5}$$

With $f_{z_2}(t) = (\dot{y} + a_4 - a_1)^2$. Since pH and temperature control is assumed, it is proposed that $\frac{da_1}{dt} = 0$, $\frac{da_2}{dt} = 0$ and it has excellent level, volume, and flow regulation $\frac{da_4}{dt} = 0$. Consequently, in this study could be the stoichiometric variable. $\frac{da_3}{dt} \neq 0$.Hence, $\widehat{\theta}_1 = \frac{a_3}{a_2 a_1}$, $\theta_2 = \frac{a_4}{a_1}$, $\theta_3 = \frac{a_4}{a_2 a_1}$ and $f_{z_2}(t) = (\dot{y} + a_4 - a_1)^2$:

$$\ddot{y} = -\widehat{\theta}_1 (\dot{y} + a_4) \exp(y) f_{z_2}(t) + \theta_2 (\dot{y} + a_4) \sqrt{f_{z_2}(t)} + \theta_3 f_{z_2}(t) u(t) + \delta(t) \tag{6}$$

Where $\phi = exp(y) f_{z_2}(t)$. Such that:

$$\ddot{y} = -\widehat{\theta}_1 \phi + \theta_2 (\dot{y} + a_4) \sqrt{f_{z_2}(t)} + \theta_3 f_{z_2}(t) u(t) + \delta(t) \tag{7}$$

The study aims for optimal biomass growth $x_{\text{ref}} = f_{\text{ref}}$ under bounded perturbations, employing a high gain filtered observer for online nutrient estimation. The algorithm outperforms similar methods in noise resilience, leveraging an optimized Riccati equation and a second-order dynamic filter (Celikovsky *et al.*,2015). This ensures rapid and stable signal processing for $x_2 = \dot{y}$, maintaining closed-loop stability. The control law design will consider discontinuities and employ sliding mode techniques, with a specified function $\mu(\cdot) = -\widehat{\theta}_1 (x_2 + a_4 - a_1)^2 \exp(x_1) + \theta_2 (x_2 + a_4)(x_2 + a_4 - a_1) + \theta (x_2 + a_4 - a_1)^2 u(t)$.

$$\begin{aligned}
\dot{x}_1 &= x_2 + h_\theta \kappa_1 \eta_1 \\
\dot{x}_2 &= \mu(\cdot) + h_\theta \kappa_2 \eta_2 \\
\dot{\eta}_1 &= -h_\theta (\eta_1 - y_1) \\
\dot{\eta}_2 &= -h_\theta (\eta_2 + h_\theta \eta_1) \\
\dot{\kappa}_1 &= h_\theta \left(-\kappa_1^2 + \kappa_1\right) \\
\dot{\kappa}_2 &= h_\theta \kappa_2
\end{aligned} \tag{8}$$

For signal output = y_1, It propose a high-gain observer for bioreactor nutrient estimation in (8), or more technical details of the above see (Robles-Magdaleno *et al.*,2020), where $h_\theta \gg 1$ is a high gain.

3 Main results

In this paper, it is proposed to change the traditional dilution rate control variable to nutrient concentration through a new mixing stage, as shown in Scheme 1. This works perfectly with our new sliding mode control method. This method works by using high-frequency modulation between the minimum and maximum values of the variable, similar to pulse width modulation (PWM). In this sense, sliding mode control has the same function. In electrical applications, a model-free PWM current control method based on digital sliding mode for switched reluctance machine drives (Rayane *et al.*, 2023) is used. The main idea behind this study is a theoretical proposal that uses the dynamic control features of the sliding mode technique. This technique is known for being accurate and strong when dealing with system uncertainties and disturbances. Importantly, such a control strategy would be infeasible with conventional dilution rates due to their less dynamic nature in response to rapid changes in system states. As a result, the switch to nutrient concentration control not only improves the system's responsiveness, but it also exemplifies the practical application of advanced control theories in optimizing bioreactor operations.

The non-linear and complex nature of biological processes makes bioreactor control difficult in biotechnology. Trajectory tracking in bioreactors keeps the process within predetermined ranges, maximizing efficiency. This section uses an adaptive sliding mode control algorithm (ASMC) for bioreactor path-following control. Sliding mode controllers can handle fluctuations, which makes them suitable for bioreactor control. The bioreactor conditions dynamically adapt the sliding mode controller. This adaptive strategy helps the controller to adapt to process changes and track the trajectory. A trajectory for the biomass concentration of microorganisms is shown below $y_P = f(t)$ and therefore $\ddot{y}_P = f''(t) = \Delta_f(t)$. The tracking error is proposed $e = y_P - y$, therefore $\ddot{e} = \Delta_f(t) - \ddot{y}$. If bioreactor dynamics is substituted in its diffeomorphic form (7):

$$\ddot{e} = \Delta_f(t) + \widehat{\theta}_1 \phi - \theta_2 (\dot{y} + a_4) \sqrt{\widehat{f_{z_2}}(t)} - \theta_3 \widehat{f_{z_2}}(t) u(t) - \delta(t) \tag{9}$$

Where $\widehat{f_{z_2}}(t)$ is an estimated function because it contains the dynamic derivative of y, which is estimated by the Filter High Gain Observer (8). Given the high-speed convergence of the high-gain observer and the discontinuity of the robust controller, the estimated function does not affect the stability of the proposed controller. The proposed sliding function is as $s = ce + \dot{e}$. Research laws determine the sliding surface and the control law for the design of sliding mode control. These fundamental research laws allow the system to reach and maintain the sliding surface, ensuring its desired behavior. It is suggested as $\dot{s} = -k_1 sgn(s)$. Therefore:

$$\begin{aligned}\dot{s} =& c\dot{e} + \Delta_f(t) + \widehat{\theta}_1\phi - \theta_2(\dot{y} + a_4)\sqrt{\widehat{f_{z_2}}(t)} \\ & - \theta_3 \widehat{f_{z_2}}(t)u(t) - \delta(t).\end{aligned} \tag{10}$$

Replacing the Research Law on the Sliding Surface

$$\begin{aligned}-k_1 \operatorname{sgn}(s) =& c\dot{e} + \Delta_f(t) + \widehat{\theta}_1\phi \\ & - \theta_2(\dot{y} + a_4)\sqrt{\widehat{f_{z_2}}(t)} - \theta_3 \widehat{f_{z_2}}(t)u(t)\end{aligned} \tag{11}$$

After numerous algebraic steps

$$\begin{aligned}u =& \left(\theta_3 \widehat{f_{z_2}}(t)\right)^{-1} (k_1 \operatorname{sgn}(s) + c\dot{e} + \Delta_f(t) \\ & + \widehat{\theta}_1\phi - \theta_2(\dot{y} + a_4)\sqrt{\widehat{f_{z_2}}(t)})\end{aligned} \tag{12}$$

If we substitute (14) in real complete with θ_1 and $\delta(t)$ (7) on dynamics tracking error the following is obtained where $\widetilde{\theta}_1 = \widehat{\theta}_1 - \theta_1$, due to the high convergence velocities of the observer (8) $f_{z_2} = \widehat{f}_{z_2}$, after numerous algebraic steps:

$$\ddot{e} = -k_1 sgn(s) - c\dot{e} - \widetilde{\theta}_1\phi + \delta(t) \tag{13}$$

This study examines a nonlinear bioreactor system with bounded perturbations and uncertainties. Thus, an adaptive and disturbance-resistant main control is achieved.

$$\begin{aligned}u =& \left(\theta_3 f_{z_2}(t)\right)^{-1} (k_1 \operatorname{sgn}(s) + c\dot{e} + \Delta_f(t) \\ & + \widehat{\theta}_1\phi - \theta_2(\dot{y} + a_4)\sqrt{f_{z_2}(t)}).\end{aligned} \tag{14}$$

With $\dot{\widehat{\theta}}_1 = s\phi$. The following theorem is proposed in order to present our main theoretical result and its proof. .

Theorem 1 Let the nonlinear bioreactor system in its diffeomorphic form shown in (7), it is known that there is an uncertainty in the parameter $\theta_1 = \frac{a_3}{a_2 a_1}$ and there is a bounded perturbation $\delta(t)$ such that $|\delta(t)| < k_1 z_1$ is satisfied. With robust control ASMC over the concentration of nutrients as the control variable (14) one has asymptotic stability.

Proof: This theorem proof requires the design of the sliding surface and the control law (7), as well as the implementation of the Lyapunov stability theory to demonstrate the stability and robustness properties. Suppose the following Lyapunov function:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\widetilde{\theta}_1^2 \tag{15}$$

one proposes the derivative under the trajectories of the previous Lyapunov function. Since $s = ce + \dot{e}$:

$$\dot{V} = s\left(\underbrace{c\dot{e} + \ddot{e}}_{\dot{s}}\right) + \widetilde{\theta}_1\dot{\widetilde{\theta}}_1 \tag{16}$$

The complete error dynamics in closed loop with control (14) is replaced by the following:

$$\dot{V} = s\left(c\dot{e} - k_1 sgn(s) - c\dot{e} - \widetilde{\theta}_1\phi + \delta(t)\right) + \dot{\widetilde{\theta}}_1 \tag{17}$$

Therefore $\dot{V} = -k_1 sgn(s)s + s\delta(t) + \widetilde{\theta}_1\dot{\widetilde{\theta}}_1 - \widetilde{\theta}_1\phi s$ where: $\dot{\widetilde{\theta}}_1 = s\phi \rightarrow \dot{\widehat{\theta}}_1 = s\phi$, if the above is fulfilled, then $\dot{V} = -k_1 sgn(s)s + s\delta(t)$. Also, it is known $|\delta(t)| < k_1 z_1 \rightarrow |\delta(t)| < k_1 z_{\max}$ implies $|\delta(t)| < k_1 z_{\max} < \varphi$, which is appropriate since every biochemical reactor has a forced steady state where it has an optimal maximal biomass. Therefore:

$$\begin{aligned}\dot{V} &= s(-k_1 sgn(s) + \varphi) \\ \dot{V} &\leq -|s|(k_1 - \varphi)\end{aligned} \tag{18}$$

Despite perturbations and parametric uncertainty, the intended trajectory is maintained because the tracking error is stable (since the error is L_2 and L_∞). Because the Lyapunov function in an experiment was a function of the sliding surface s and the parameterized error $\widetilde{\theta}_1$, it should be noted that at the conclusion of the test, only s is discovered. It is possible to ensure the stability of L_2 and L_∞ using Barbalat's theorem and the classical adaptive control test; this analysis has been omitted for brevity. For more details, see (Aguilar-Ibanez *et al.*,2021).

4 Numerical results

This section discusses the numerical results of the ASMC bioreactor control. Scientific simulations of Matlab Simulink 2020 b dynamic systems yielded these results. To analyze the behavior of bioreactors under unknown conditions and parameters, control systems need a numerical simulation of $\widehat{\theta}_1$ and the presence of disturbance $\delta(t)$. Through numerical simulation, the performance of ASMC (14) was compared to that of the classical PID control. The results of the numerical simulation will be detailed below. Adaptive sliding mode control improves the productivity and efficiency of the bioreactor. The use of nutrient concentration as a control variable and the proposed dilution rate as a constant parameter to help establish robust control is also examined. The following parameters of the controllers and observed are as follows based on (Celikovsky *et al.*,2015) . The biomass output signals Gaussian noise simulates the behavior of the bioreactor sensor.

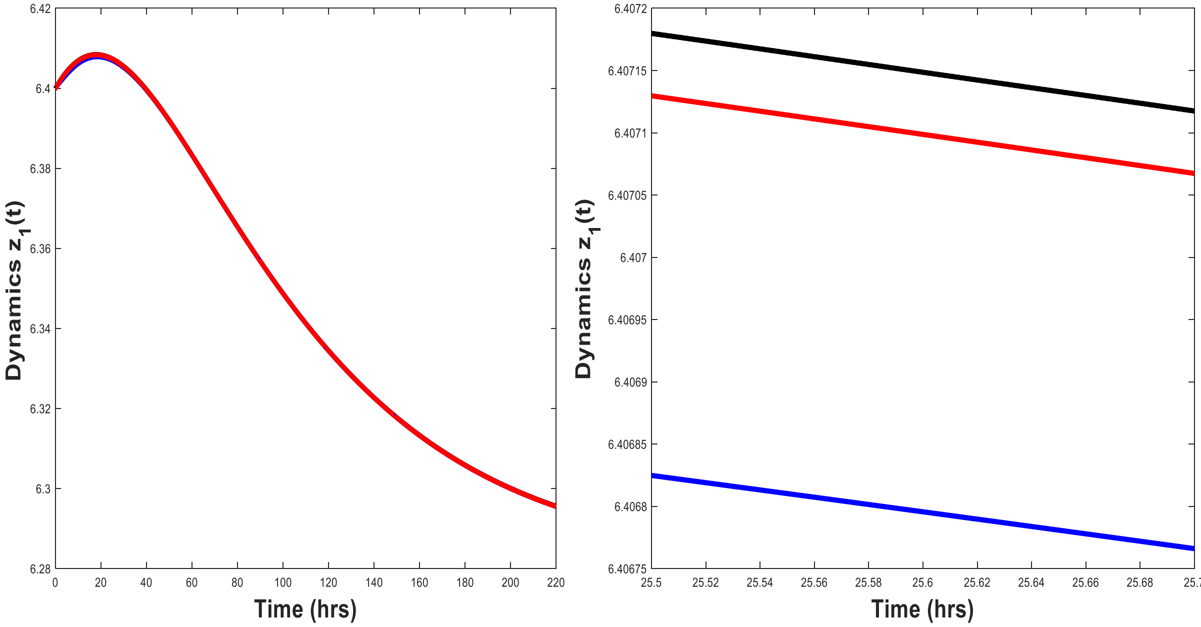

Figure 2: Biomass dynamics under PID (blue) and ASMC (black) controls over 220 hours and a detailed view from 25.5 to 25.7 hours. The reference trajectory is depicted in red.

Table 1. Parameters of simulation.

Parameter	Value	Description
a_1	0.027	Maximum rate (h^{-1})
a_2	25	Saturation (mg/L)
a_3	3.45	Constant stoichiometry (- -)
a_4	0.018	Dilution rate (h^{-1})
$x_1(0)$	600	Biomasss initial (mg/L)
$x_2(0)$	60	Nutrients initial (mg/L)
$z_1(0)$	6.4	Initial diffeolmorphic (mg/L)
$z_2(0)$	0.001	Initial diffeolmorphic (h^{-1})

Turbidity-based sensors in practical applications often have noise issues, making this adjustment crucial. Noise mimics these sensors' operational variability and inaccuracies. Recent academic research emphasizes sensor noise in simulation models to improve control strategy predictive accuracy and reliability (Linets *et al*., 2023).

The proposed method is a Dorman-Prince numerical method for ode08 with a step size of 0.01. To make the simulation more plausible from a practical standpoint, it is proposed to saturate the control signal from 0 to 1000 mg/L and to make the transitions as seamless as possible by passing the signal through a first-order filter before injecting it into the actuators. On follows law control:

$$u = \left(0.02667 f_{z_2}(t)\right)^{-1} \left(sgn(s) + 1000\dot{e} + \Delta_f(t) + \widehat{\theta}_1 \phi - 0.667(\dot{y} + 0.018)\sqrt{f_{z_2}(t)}\right), \quad (19)$$

$$\dot{\widehat{\theta}}_1 = s\phi,$$
$$s = 1000e + \dot{e},$$
$$f_{z_2}(t) = (\dot{y} - 0.009)^2$$

Where $\Delta_f(t) = \ddot{y}_p$ that is the second derivative of the tracking equation, which is exactly the same as the sample plant except that the parameter a_3 is 3.45 which is equivalent to 13.76 % more parametric perturbation. Also it there is $\delta(t) = 0.00002 z_1 * sin(10t)$ perturbation on our plant to be controlled. Whit $h = 100$, $l_1 = 10$ and $l_2 = 10$ for high gain observer (8).The plant simulated is:

$$\ddot{y} = -\widehat{\theta}_1(\dot{y} + 0.018)\exp(y) f_{z_2}(t) + 0.667(\dot{y} + 0.018)\sqrt{f_{z_2}(t)} + 0.02667 f_{z_2}(t) u(t) + \delta(t) \quad (20)$$

$$\delta(t) = 0.00002 z_1 * \sin(10t)$$

The present study compares a PID controller optimized using the Ziegler-Nichols tuning method with an advanced sliding mode control strategy proposed by ASMD. The Ziegler-Nichols method facilitated the configuration of a fundamental controller, serving as a crucial foundation for the subsequent comparative analysis. This well-established technique successfully optimized the proportional-integral-derivative (PID) settings of

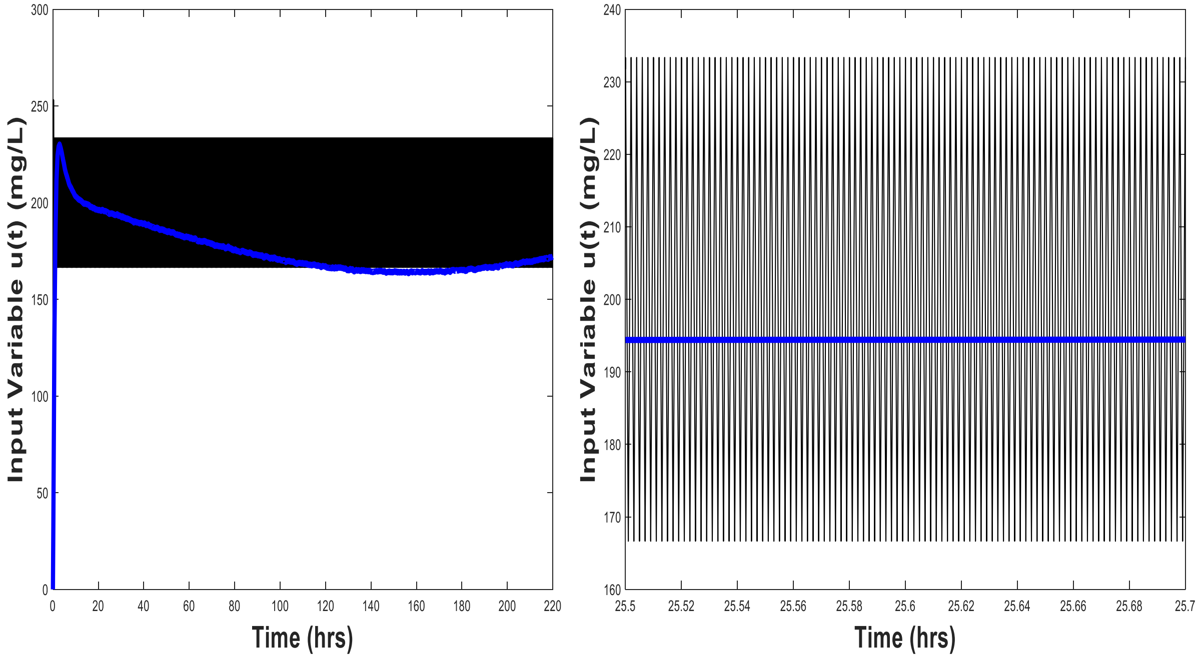

Figure 3: Control variable responses for ASMC (black) and PID (blue) across the full timeline and a zoomed interval from 25.5 to 25.7 hours.

the linearized system by systematically evaluating the two control strategies using a conventional but robust approach. The analysis primarily examined several performance indices, including the integral of quadratic error (ISE), the integral of time-weighted absolute error (ITAE), the integral of time-weighted quadratic error (ITSE), the integral of absolute error (IAE), and the integral of quadratic error multiplied by time (ITSE), in the sence (Tomic et., 2013). The evaluation framework played a crucial role in measuring the effectiveness and efficiency of PID control, tuned using the Ziegler-Nichols method, in controlling the system's dynamic responses under different operating conditions. The comprehensive evaluation aimed to compare the performance of PID and sliding mode control strategies, focusing on their capabilities to minimize specific error metrics within the control system environment.

4.1 *Discussion of the results*

An important aspect of our study is that instead of the traditional dilution rate, we used nutrient concentration as the control variable. This was made possible by using a high-frequency mixer. This novel method enables accurate and swift modifications in nutrient levels, thereby improving the responsiveness and stability of the control system. The utilization of nutrient concentration as a control variable is crucial for the theoretical progress of reactor control through high-frequency techniques. The ASMC method demonstrates both feasibility and a notable enhancement in control performance, as indicated by the lower error indices achieved in comparison to the PID method. Additionally, Figure 1 illustrates how ASMC provides a better approximation to the reference trajectory compared to PID, further highlighting the effectiveness of our approach.

Figure 3 depicts the control variables responses to the Adaptive Sliding Mode Control (ASMC) and Proportional-Integral-Derivative (PID) methods over a period of 220 hours. The high-frequency oscillatory behavior observed in the ASMC control variable, commonly known as "chattering," is a distinctive feature of sliding mode controllers. This behavior is a result of ASMC inherent design, which includes quick switching to ensure system stability and resilience in the face of uncertainties and disturbances. To get a full picture of the performance, we changed the Integral of Time-weighted Absolute Error (ITAE), the Integral of Squared Error (ISE), and the Integral of Absolute Error (IAI) indices by multiplying them by 520, 50,000, and 300, respectively. It is taken this action to enhance the visual representation of the results. It is evaluated the scaled values and found that the ITAE (Integral of Time multiplied by Absolute Error) for the PID (Proportional-Integral-Derivative) controller was 177.32 (0.341 multiplied by 520), while it was 0.546 for the ASMC (Active Set Model Control). Similarly, we calculated the integral square error (ISE) values as 0.073 (1.46e-06 multiplied by 50,000) for the proportional-integral-derivative (PID) method and 7.94e-06 for the adaptive sliding mode control (ASMC) method. The IAI obtained a PID value of 0.0056 (calculated as 1.86e-02 multiplied by 300), while the ASMC value was 0.0188.

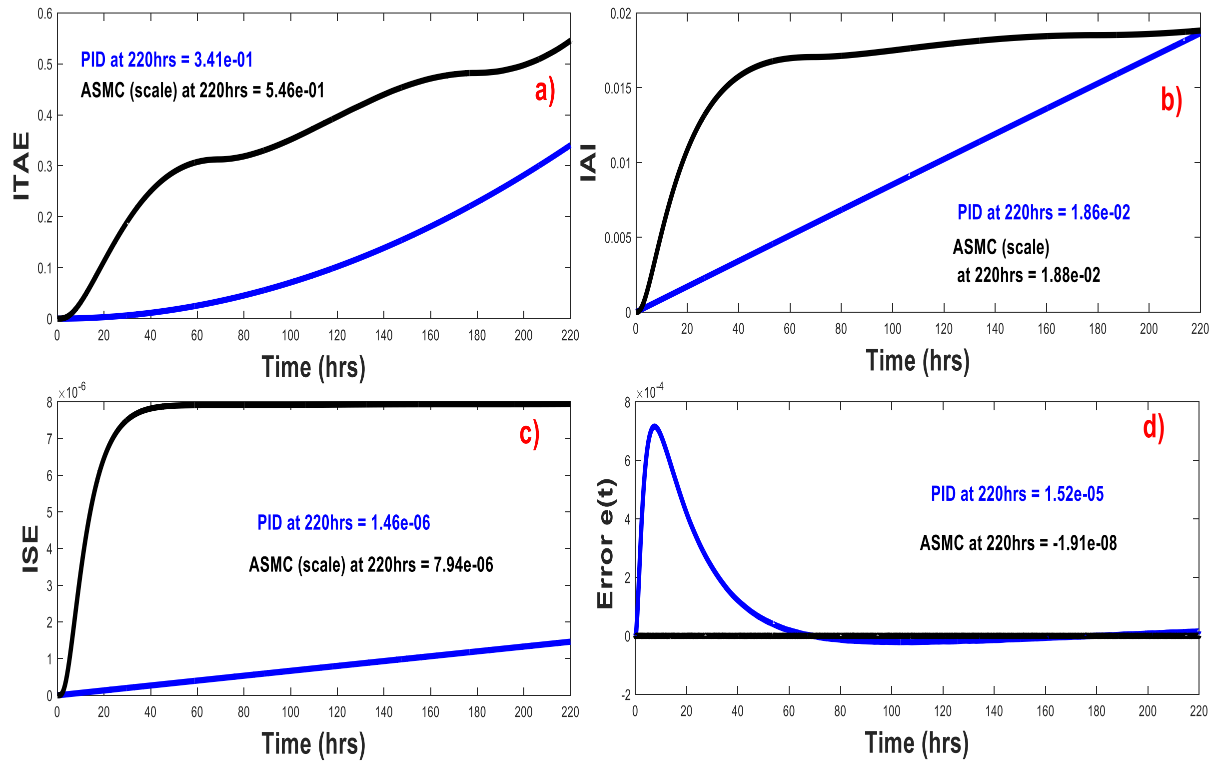

Figure 4: Comparison of performance indices for PID (blue) and ASMC (black) controls. (a) ITAE showing total time-weighted absolute error, where the PID value, after applying the scaling factor, is 177.32 (originally 0.341 without scaling), and the ASMC value is 0.546 (without scaling). (b) IAI focusing on cumulative error, where the PID value, after applying the scaling factor, is 0.0056 (originally 1.86e-02 without scaling), and the ASMC value is 0.0188 (without scaling). (c) ISE emphasizing squared error severity, where the PID value, after applying the scaling factor, is 0.073 (originally 1.46e-06 without scaling), and the ASMC value is 7.94e-06 (without scaling). (d) Dynamic error e(t), comparing the performance of both control methods over 220 hours. Each metric evaluates control performance up to 220 hours.

Although there may be some chattering, the ASMC exhibits exceptional precision and resilience, especially when dealing with nonlinear dynamics and disturbances, respectively, as illustrated in Figure 4. Both strategies showed similar performance in terms of IAI, with insignificant discrepancies between their scores (PID: 1.86e-02, ASMC: 1.88e-02), suggesting comparable error levels over time. less strict conditions.

An important aspect of our study is that instead of the traditional dilution rate, we used nutrient concentration as the control variable. It is made this possible by using a high-frequency mixer. This novel method enables accurate and swift modifications in nutrient levels, thereby improving the responsiveness and stability of the control system. The utilization of nutrient concentration as a control variable is crucial for the theoretical progress of reactor control through high-frequency techniques. The ASMC method demonstrates both feasibility and a notable enhancement in control performance, as indicated by the lower error indices achieved in comparison to the PID method.

As shown in Figure 4. The scaling factors for ASMC control are as follows: ITAE * 520, ISE * 50,000, and IAI * 300. The results clearly demonstrate that, despite the inherent chattering effect in ASMC, it achieves significantly lower error indices compared to the PID controller, showcasing its superior accuracy and robustness in managing nonlinear dynamics and disturbances.

The end result of both control strategies was the same in the diffeomorphic variable 1x1, which shows the natural logarithm of biomass concentration (Figure 2). The alignment with the reference trajectory demonstrates the control's optimal performance under saturated conditions. On the other hand, the ASMC showed a higher level of control input (2.33e+02 vs. 1.72e+02), which means it made more responsive and forceful changes to keep the system stable and track its path, especially during temporary conditions seen between 25.5 and 25.7 hours (Figure 3).

The comparison of PID and ASMC in bioreactor operation highlights the importance of choosing a suitable control strategy that aligns with specific operational objectives and environmental circumstances. PID control offers a smoother and more effective way to minimize errors in standard operations. On the other hand, ASMC provides increased robustness against uncertainties in parameters and disturbances, which is crucial for maintaining stability in complex or unstable environments. This analysis not only aids in the continuous improvement and refinement of control systems for bioreactors, but it also makes a substantial contribution to the understanding of adaptive control strategies in dynamic systems that encounter real-world variations and uncertainties. ASMC is considered a superior strategy, even though it has some inherent noise and practical limitations. However, a well-optimized PID system also performs well under less strict conditions.

Finally, assumption where the position tracking is not one of reaching sliding surface, but one of near or already on sliding surface can be made. However, an ASMC is basically a roust control technique which is used to control systems with uncertainties. In general comparison between any two controllers should be based on certain particular cases, because one controller may not be better in all circumstances. Therefore, a robust PID controller may provide a similar/comparable response as an SMC in certain cases. Having said that, the biggest advantage an ASMC possess over PID is the finite time stability of the closed loop system (e.g. may not be true for discrete time systems). Another important point to remember is that the ASMC can reject the disturbances (disturbance decoupling, when properly designed), whereas PID can reduce or remove the effect of uncertainties.

Despite the fact ASMC offers some advantages over PID control, it also has some boundaries and tests, such as chattering, control signal discontinuities, and sensitivity to noise. The choice between ASMC and PID control depends on the detailed necessities of the control problem, the system dynamics, and the trade-offs between performance, robustness, and complexity.

Conclusion

It was found that adaptive sliding mode control (ASMC) with a new diffeomorphic transformation is better at controlling the amount of nutrients in a bioreactor than traditional PID control based on the ITAE, ISE and IAI error rates, but the presented controller presents the classical chattering control over the control variable, but being the concentration variable, it can mix quickly to reach the required dynamics. It effectively handled the non-linear and intricate dynamics of the system, even in the presence of uncertainties and disturbances. The implementation of a high-gain filter observer greatly improved nutrient concentration estimation, thus increasing the controller's effectiveness. These findings underscore the potential of ASMC to enhance bioreactor operations, especially in systems where a constant dilution rate regulates nutrient concentration. But it was also demonstrated that the classical PID controller tuned with the method shown maintains a very high dynamic performance. This represents a significant advancement in bioprocess control. Subsequent investigations will aim to confirm these findings in extensive bioreactors and broaden the application of this sophisticated control approach to additional fermentation procedures.

Nomenclature

x_1	Biomass concentration within the bioreactor (mg/L).
x_2	Nutrient concentration within the bioreactor (mg/L).
a_1	Maximal growth rate of the microorganisms (h^{-1}).
a_2	Saturation constant, indicative of the biomass's affinity for the nutrient (mg/L).
a_3	Stoichiometric constant, delineating the stoichiometric relationship between biomass consumption and nutrient uptake (-).
a_4	Dilution rate, representing the rate at which fresh medium is introduced into the bioreactor (h^{-1}).
$u(t)$	Control variable denoting the concentration of nutrients at the reactor's inlet (mg/L).
$\mu(\cdot)$	Specific growth rate of microorganisms as a function of nutrient concentration.
$\delta(t)$	Symbolizes uncertainties and bounded perturbations influencing the bioreactor system.
$\theta_1, \theta_2, \theta_3$	Parameters in the transformed dynamical model for the bioreactor control system.
$\hat{\theta}_1$	Estimated parameter of θ_1 within the controller.
z_1, z_2	State variables derived from diffeomorphism, employed in the equations of the transformed control system.
$f_{z_2}(t)$	Function associated with the transformed state variable z_2, utilized in the control system equations.
ϕ	Derived variable representing a composite of state variables and parameters within the transformed control system equations.
s	Sliding function in adaptive sliding mode control.
k_1, c	Gain constants in the sliding mode control algorithm, where k_1 is the control law gain and c is the sliding function gain.
h_θ, l_1, l_2	Parameters of the high-gain observer used for nutrient estimation, where h_θ signifies the high gain, and l_1 and l_2 are observer gains.

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