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NONLINEAR OBSERVER FOR SWITCHED SYSTEMS: APPLICATION TO A BATCH BIOREACTOR

OBSERVADOR NOLINEAL PARA SISTEMAS CONMUTADOS: APLICACIÓN A UN BIORREACTOR EN LOTE

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Abstract
The main objective of this work is to design a new state observer for a switched nonlinear system. The proposed observer contains a standard proportional term and a bounded feedback that improves the performance of the estimation process compared with other methodologies such as extended Luenberger-type observers. A theoretical analysis of the convergence of the proposed observer through an analysis of multiple Lyapunov functions is provided. The estimation strategy is applied to a bioreactor batch, where is performed a process of sulfate-reducing considering a hypothetical change in kinetic regime. The evolution of the reaction kinetics is described in three stages: the first stage is described by the Haldane-Boulton kinetic model; the second transitional stage called an interaction between the Haldane-Boulton and Moser-Boulton kinetic models; and the third stage is represented by the model Moser-Boulton. Sulfide concentration is considered as a measured variable of the bioreactor to implement the proposed observer. Numerical experiment results shown satisfactory performance of the considered methodology.

Keywords: nonlinear observer, switched system, adaptive modeling, batch bioreactor.

Resumen
El objetivo principal del trabajo es diseñar un nuevo observador de estados para un sistema no lineal conmutado. El observador propuesto contiene un término proporcional estándar y una retroalimentación acotada que mejora el desempeño del proceso de estimación en comparación con otras metodologías, como los observadores de tipo Luenberger extendidos. Se proporciona un análisis teórico sobre la convergencia del observador propuesto a través de un análisis de funciones múltiples de Lyapunov. La estrategia de estimación es aplicada a un biorreactor en lote, donde se lleva a cabo un proceso de sulfato-reducción considerando un cambio hipotético de régimen cinético. La evolución de la cinética de reacción es descrita en tres etapas: la primera etapa es descrita por el modelo cinético de Haldane?Boulton; la segunda etapa llamada de transición se asume una interacción entre el modelo de Haldane-Boulton y el modelo cinético Moser-Boulton; y la tercera etapa es representada por el modelo de Moser-Boulton. La concentración de sulfuro se considera como una medida de la salida del biorreactor para implementar el observador propuesto. Los resultados de los experimentos numéricos muestran un desempeño satisfactorio de la metodología considerada.

Palabras clave: observador no lineal, Sistema cambiante, Modelo adaptable, Biorreactor por lote.

1 Introduction

The problem with the complexity of dynamic nonlinear systems appears in a great number of scientific and engineering domains. Some decomposition and simplification techniques were developed in the last years to make a complexity reduction, according to objectives like identification,

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controller design or stability analysis. Simulations on mathematical model of actual bioreactor runs suggest how process variables, such as substrate and product concentrations, change and how nutrient feeding should be tuned-in to time, concentration, and composition to reach a desired response. Insights gained from modeling can guide us in the adjustment of a process, reducing the number of characterization rounds required. Furthermore, comparing actual experimental results with model predictions helps to improve the models themselves. It is important to note that outputs can vary in unpredictable ways if processes are simulated outside boundaries set by the models that describe them, especially if the real operating ranges of actual processes are obtained inadequately (Dang *et al*., 2011). As these models are more complex, they can correctly describe the interactions between microorganisms, proteins and substrates in a reactor, but sampling on-line key process variables is often the problem and usually there is missing information until the samples are properly processed and analyzed (Yuan *et al*, 2008). Thus, the named *state observers* have been a successful way to predict these non-measurable process variables. Several observers' structures have been proposed to cover different requirements, also presenting different mathematical structures (Alvarez and Simutis, 2004).

Although the *state observers* may give the missing information, the model may still lack in terms of explaining when the system has external perturbations, forcing a change in the kinetic behavior of the microorganism contained in the reactor. When an external perturbation (eg. change in temperature, pH, a new electron donor/acceptor, agitation, aeration (Hamilton *et al*., 2005) is present and the model, which is usually made-up of standard fixed kinetic models, cannot follow the new kinetic behavior and it will eventually lead to wrong predictions or inadequate process control. A non-conventional way to deal with the above is to use models with switchable structures, which are a part of the *hybrid dynamical* systems (Brandt *et al*., 2004; Dang *et al*., 2011).

Hybrid dynamical systems are those that involve both continuous and discrete dynamics (Branicky *et al*., 1998). The switching systems, are a class of hybrid dynamical systems, where the underlying model is a continuous model, with an event triggering a switch (switching signal) (Liberzon and Morse, 1999). Traditionally, most of the research done with the switching systems, have been with dynamical systems that are described purely as either time-driven continuous variable dynamics or event-driven discreet logic dynamics (Branicky, 1998; Tartakovsky *et al*., 2002; Aguilar-Garnica *et al*., 2009; Gouzé and Sari, 2010). The switching systems have been identified in a wide variety of natural and man-made systems, for example gene regulatory networks, biological processes, embedded systems, process control, communication networks, aircraft and traffic control, and among many other fields (Branicky *et al*., 1998).

State observers have been designed for a class of switched systems and have been applied to linear switched systems, as mentioned in several papers published in the open literature, in which, observers were designed for uncertain systems, time-delay and noisy measurements (Alessandri and Coletta, 2001). For nonlinear switched systems, several approaches considering, proportional, sliding-modes, fragile observers, high gain and so on, were considered in Liu (1997), with application to electrical and mechanical systems being the most frequent, however nonlinear observer's designs, in particular the application related with biological reactors are not as large as the others.

From the above, in this work is proposed a nonlinear observer to estimate the state variables of the observable subspace in a sulfate-reducing bioreactor, considering that the sulfide concentration is the measured variable. *Desulfovibrio alaskensis* 6SR is used as model from sulfate-reducing process (Neria-González *et al*., 2006), which was modeled as a nonlinear switched system with two different kinetic regimes due to a hypothetical change in the metabolic pathways.

2 Bioreactor modeling

2.1 Mathematical support

In this work the simplest model of a nonlinear system with a discontinuity on the right hand side will be considered as follows (Dieci and Lopez, 2009):

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) = \begin{cases} \mathbf{f}_1(\mathbf{x}(t)), \mathbf{x} \in S_1 \\ \mathbf{f}_2(\mathbf{x}(t)), \mathbf{x} \in S_2 \end{cases} \qquad \mathbf{x}(0) = x_0 \in \mathbb{R}^n \tag{1}$$

The state space $\mathbb{R}^n$ is split into two subspaces S_1 and S_2, leading to a discontinuity in each dynamic system (Bernardo *et al*., 2008).

The Filippov convex method is applied to turn the discontinuous system (1) into a convex differential

inclusion. Letting the derivate of the solution of the switched system to be contained in a compact subset of the continuous function set, and therefore allowing the existence of continuous differentiable solutions. The existence of solutions is guaranteed as a notion of upper semi-continuity of set-valued functions (Dieci and Lopez, 2009; Machina and Ponosov, 2011).

Considering the above the system (1) becomes a switched system:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t)) = \begin{cases} \mathbf{f}_1(\mathbf{x}(t)), & \mathbf{x} \in S_1 \\ \overline{co}\{\mathbf{f}_1(\mathbf{x}(t)), \mathbf{f}_2(\mathbf{x}(t))\}, & \mathbf{x} \in \Sigma \\ \mathbf{f}_2(\mathbf{x}(t)), & \mathbf{x} \in S_2 \end{cases} \tag{2}$$

Where $\overline{co}\{\mathbf{f}_1(\mathbf{x}(t)), \mathbf{f}_2(\mathbf{x}(t))\}$, denotes the smallest closed convex set, as:

$$\overline{co}\{\mathbf{f}_1(\mathbf{x}(t)), \mathbf{f}_2(\mathbf{x}(t))\} \tag{3}$$
$$= \{\mathbf{f}_v \in \mathbb{R}^n : \mathbf{f}_v = (1-\alpha)\mathbf{f}_1(\mathbf{x}(t)) + \alpha\mathbf{f}_2(\mathbf{x}(t)), \alpha \in [0,1]\}$$

To achieve the solution the system's state space is divided into two subspaces or stages (S_1 and S_2) by Σ, an hypersurface, such that $\mathbb{R}^n = S_1 \cup \Sigma \cup S_2$. The hypersurface is defined by a scalar indicator (or event) functiong: $\mathbb{R}^n \to \mathbb{R}$, so that the subspaces S_1 and S_2, and the hypersurface Σ, are characterized as:

$$\Sigma = \{\mathbf{x} \in R^n | g(\mathbf{x}) = 0\},\ S_1 = \{\mathbf{x} \in R^n | g(\mathbf{x}) < 0\},$$
$$S_2 = \{\mathbf{x} \in R^n | g(\mathbf{x}) > 0\} \tag{4}$$

2.2 Kinetic modeling

The considered hybrid model is applied to an anaerobic bioreactor operating in batch, where a sulfate-reducing process takes place, considering as the state variables sulfate, biomass and sulfide mass concentrations. It is assumed a kinetic regimen change due to an unknown external perturbation, where the bioreactor dynamics are described by the unstructured kinetic model Haldane-Boulton, since the sulfate-reducing bacteria is inhibited by substrate and product in a first stage, then a transition stage in the kinetic regimen is assumed, caused by an unknown perturbation in the system and represented by the interaction between the Haldane-Boulton model and the Moser-Boulton kinetic models, both models consider inhibition by substrate and product, but the perturbation in the system benefits the bacteria in the way that more sulfate is consumed, thus more biomass and sulfur is produced and the final stage where the unstructured kinetic model Moser-Boulton the only present, this model, as stated before, considers substrate and product inhibition, but this inhibition minor compared to one with Haldane-Boulton kinetic model.

The Haldane-Boulton and the Moser-Boulton kinetic models where individually corroborated with experimental data (Neria-González *et al*., 2011), showing a satisfactory performance in accordance with linear correlation coefficient criteria. The Haldane-Boulton model is given by the following equation:

$$\boldsymbol{\mu_{HB}} = \left[\frac{\mu_m S}{K_S + S + S^2 K_i^{-1}}\right]\left[\frac{K_P}{K_P + P}\right] \tag{5}$$

And the Moser-Boulton model is described by the following equation:

$$\boldsymbol{\mu_{MB}} = \left[\frac{\mu_m S^n}{K_S + S^n}\right]\left[\frac{K_P}{K_P + P}\right] \tag{6}$$

The corresponding sets of parameters are given in Tables 1 and 2.

Finally the switching model is represented as follows:

$$\begin{bmatrix} \dot{\mathbf{S}} \\ \dot{\mathbf{X}} \\ \dot{\mathbf{P}} \end{bmatrix} = \begin{cases} \overline{\mu}_1 \mathbf{X}; & \mathbf{x} \in S_1 \\ (1-\alpha)\mathbf{X} + \alpha\left(\overline{\mu}_2\right)\mathbf{X}; & \mathbf{x} \in \Sigma \\ \overline{\mu}_2 \mathbf{X}; & \mathbf{x} \in S_2 \end{cases} \tag{7}$$

Where:

$$\overline{\mu}_1 = \begin{bmatrix} \frac{\mu_{HB}}{Y_{S/X}} \\ \left(\mu_{HB} - \mu_d\right) \\ \frac{\mu_{HB}}{Y_{P/X}} \end{bmatrix};\ \overline{\mu}_2 = \begin{bmatrix} \frac{\mu_{MB}}{Y_{S/X}} \\ \left(\mu_{MB} - \mu_d\right) \\ \frac{\mu_{MB}}{Y_{P/X}} \end{bmatrix}$$

S, **X** and **P** represent the sulfate, biomass and sulfide concentrations respectively, $Y_{S/X}$ and $Y_{P/X}$ are the yield coefficients, μ_d is the death kinetic constant, μ_m is the maximum specific cell growth rate, related with the maximum biomass concentration reached in a batch bioreactor, K_s is named the affinity constant, this parameter is related with the *affinity* of the microorganism to the substrate, K_i is the substrate inhibition constant, K_p is the product inhibitions constant, n is the order of the biochemical reaction and α is a weighting vector parameter.

Table 1. Parameters values

Kinetic model	$\mu_m(h^{-1})$	K_s (mg/L)	K_i (mg/L)	K_p (mg/L)	n
Moser-Boulton	10.55	1.26E+09	-	2.70	2.53
Haldane-Boulton	39.84	86070.00	9850.19	7.24	-

Table 2. Experimental Initial conditions and Parameters values

Parameter	Value
Initial Biomass concentration (mg/L)	134.73
Initial Sulfate concentration (mg/L)	5057.47
Initial Sulfur concentration (mg/L)	39.56
$Y_{(S/X)}$ (mg S mg^{-1} X)	14.13
$Y_{(P/X)}$ (mg P mg^{-1} X)	2.14
μ_d (h^{-1})	0.0058

3 Local observability analysis and observer design

3.1 Local observability

The preceding problem to observer design is to analyze the observability conditions of the nonlinear systems under study. For linear systems, classical observability index as observability matrix for observability analysis and the estimator design have been extensively studied, and have proven extremely useful, especially for on-line monitoring and control applications such as observer based control design.

The design of observability conditions for nonlinear systems is a challenging problem (even for accurately known systems) that has received a considerable amount of attention. A first category of techniques consists in applying linear algorithms to the system linearized around the estimated trajectory.

Now, in order to prove the local observability of a switched model, a linearization of the system under study must be done to make the corresponding analysis (Ezzine and Haddad, 1998; Vidal *et al*., 2002). The system (2) can be linearized trough Taylor series, so the local observability analysis can be applied to the linearized system given by equation (8).

$$\dot{\mathbf{x}}(t) \in \mathbf{J_F}\mathbf{F}(\mathbf{x}(t)) = \begin{cases} \mathbf{J_F}(\mathbf{f}_1(\mathbf{x}(t)));\ \mathbf{x} \in S_1 \\ \mathbf{J_F}((1-\alpha)\mathbf{f}_1(\mathbf{x}(t)) + \alpha\mathbf{f}_2(\mathbf{x}(t)));\ \mathbf{x} \in \Sigma \\ \mathbf{J_F}(\mathbf{f}_2(\mathbf{x}(t)));\ \mathbf{x} \in S_2 \end{cases} \tag{8}$$

Or the equivalent form:

$$\dot{\mathbf{x}}(t) \in \mathbf{J_F}\mathbf{F}(\mathbf{x}(t)) = \begin{cases} \mathbf{J_{F1}}\mathbf{x}; \mathbf{x} \in S_1 \\ \mathbf{J_{F2}}\mathbf{x}; \mathbf{x} \in \Sigma \\ \mathbf{J_{F3}}\mathbf{x}; \mathbf{x} \in S_2 \end{cases}$$

With a linear measured vector:

$$\mathbf{y} \in C\mathbf{x}(t) = \begin{cases} C_1\mathbf{x} \\ \vdots \\ C_p\mathbf{x} \end{cases}$$

Where $\mathbf{J_F}$ is the corresponding Jacobian matrix of the system (3) and p is the number of subsystems.

The observability analysis for switched linear or linearized dynamic systems is a natural extension of the well-known observability analysis for standard linear systems, which generally is represented by the observability matrix (Sun and Ge, 1968; Vidal *et al*., 2002; Babaali and Pappas, 2005). This observability matrix for switched linear and/or linearized systems can be defined by the following definition:

Definition 1. (See Sun and Ge, 1968). A class continuous-time switched linear dynamic system is path-wise observable if and only if their corresponding dynamic subsystems are completely observable. Therefore, the corresponding observability matrix for a class of switched continuous-time linear system is expressed as shown in Eq. (9).

Similar to standard linear systems, it is required that the observability matrix $\mathbf{Q}$ be full rank, i.e. *rank* $[\mathbf{Q}] = n$, in order to provide full state observability. This results is applied in section 4 to analyze the local observability properties of the switched bioreactor's model.

$$\mathbf{Q}=\left[\begin{array}{c} C_1,\cdots,C_p,C_1\mathbf{A}_1,\cdots,C_p\mathbf{A}_1,\cdots,C_1\mathbf{A}_p,\cdots,C_p\mathbf{A}_p,\cdots,C_1\mathbf{A}_1^2,\cdots,C_1\mathbf{A}_1\mathbf{A}_p,\cdots, \\ C_p\mathbf{A}_1\mathbf{A}_p,\cdots,C_1\mathbf{A}_1^{n-1},\cdots,C_1\mathbf{A}_p\mathbf{A}_1^{n-2},\cdots,C_p\mathbf{A}_p\mathbf{A}_1^{n-2},\cdots,C_p\mathbf{A}_p^{n-1} \end{array}\right]^T \tag{9}$$

3.2 Observer design

Let us consider the switched bioreactor's model in a generalized state space form:

$$\dot{\mathbf{x}}(t)\in\mathbf{F}(\mathbf{x},\mathbf{u})=\begin{cases}\mathbf{f}_1(\mathbf{x},\mathbf{u}); & \mathbf{x}\in S_1\\ (1-\alpha)\mathbf{f}_1(\mathbf{x},\mathbf{u})+\alpha\mathbf{f}_2(\mathbf{x},\mathbf{u}); & \mathbf{x}\in\Sigma\\ \mathbf{f}_2(\mathbf{x},\mathbf{u}); & \mathbf{x}\in S_2\end{cases} \tag{10}$$

$$\mathbf{y}=C\mathbf{x}$$

Here $\mathbf{x}\in\mathbb{R}^n$ is the state vector, which takes values in $\mathbf{X}$ as a connected manifold of dimension n, $\mathbf{u}\in\mathbb{R}^q$ is the vector of external control inputs, taking values in some open subset U, finally $\mathbf{y}\in\mathbb{R}^m$ describe the vector of measured outputs taking values in some open subset $\mathbf{Y}$. The function f_i will be considered as a smooth function (C^∞) of their arguments, and input functions $u^{(o)}$ to be locally bounded and measurable functions in a set U.

Then, for the state space model given by (10), it is proposed a state variables observer, in accordance with Proposition 1, as:

Proposition 1. The following dynamic system is a state variable observer of system (10):

$$\dot{\hat{\mathbf{x}}}=\begin{cases}\mathbf{f}_1(\hat{\mathbf{x}},\mathbf{u})+\mathbf{k}_p sign*\frac{\varepsilon^n}{1+\varepsilon^n}+\mathbf{k}_q\varepsilon;\ \hat{\mathbf{x}}\in S_1\\ (1-\alpha)\mathbf{f}_1(\hat{\mathbf{x}},\mathbf{u})+\alpha\mathbf{f}_2(\mathbf{x},\mathbf{u})+\mathbf{k}_p sign*\frac{\varepsilon^n}{1+\varepsilon^n}+\mathbf{k}_q\varepsilon;\ \hat{\mathbf{x}}\in\Sigma\\ \mathbf{f}_2(\hat{\mathbf{x}},\mathbf{u})+\mathbf{k}_p sign*\frac{\varepsilon^n}{1+\varepsilon^n}+\mathbf{k}_q\varepsilon;\ \hat{\mathbf{x}}\in S_2\end{cases} \tag{11}$$

Where the estimation error is defined as:

$\varepsilon=\mathbf{x}-\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}$ is the vector of estimated state variables.

Sketch of proof of Proposition 1.

By considering equations (10) and (11), the dynamic differential equation of the estimation error ($\dot{\varepsilon}=\dot{x}-\dot{\hat{x}}$) is given by

$$\dot{\varepsilon}=\begin{cases}\mathbf{f}_1(\mathbf{x},\mathbf{u})-\mathbf{f}_1(\hat{\mathbf{x}},\mathbf{u})-\mathbf{k}_p sign*\frac{\varepsilon^n}{1+\varepsilon^n}-\mathbf{k}_q\varepsilon;\ \hat{\mathbf{x}}\in S_1\\ (1-\alpha)(\mathbf{f}_1(\mathbf{x},\mathbf{u})-\mathbf{f}_1(\hat{\mathbf{x}},\mathbf{u}))-\alpha(\mathbf{f}_2(\mathbf{x},\mathbf{u})-\mathbf{f}_2(\hat{\mathbf{x}},\mathbf{u}))-\mathbf{k}_p sign*\frac{\varepsilon^n}{1+\varepsilon^n}-\mathbf{k}_q\varepsilon;\ \hat{\mathbf{x}}\in\Sigma\\ \mathbf{f}_2(\mathbf{x},\mathbf{u})-\mathbf{f}_2(\hat{\mathbf{x}},\mathbf{u})-\mathbf{k}_p sign*\frac{\varepsilon^n}{1+\varepsilon^n}-\mathbf{k}_q\varepsilon;\ \hat{\mathbf{x}}\in S_2\end{cases} \tag{12}$$

In order to prove stability of the estimation error for the system (12), a Multiple Lyapunov Functions analysis was applied (Branicky, 1998; Zhang *et al*., 2013). Thus, for the considered system (12), Lyapunov ($\mathcal{L}_i$) functions exist, which have to be continuous positive-definite with continuous partial derivatives, when valued in zero the function is, at the origin, ($\mathcal{L}_i(0)=0$) for each considered subsystem of the proposed system throughout each period of time in which the subsystem takes place, and the derivate of said functions has to be negative semi-definite ($\dot{L}(x(t))\leq 0$) for that same periods of time.

Therefore, if there is a set of Lyapunov-type functions that matches the number of switches that the proposed switched system has, and the proposed Lypunov-type functions cover all the criteria previously mentioned, the proposed system is stable in the Lyapunov sense (Li *et al*., 2013).

Now, consider the following Lyapunov candidate function:

$$\dot{\mathcal{L}}\in\left\{\begin{array}{c}\dot{\mathcal{L}}_1\\ \dot{\mathcal{L}}_{1,2}\\ \dot{\mathcal{L}}_2\end{array}\right.\Rightarrow\varepsilon^T P\varepsilon\in\|\varepsilon\|_P^2\in\left\{\begin{array}{c}\|\varepsilon_1\|_P^2\\ \left\|\varepsilon_{1,2}\right\|_P^2\\ \|\varepsilon_2\|_P^2\end{array}\right.,P=P^T>0 \tag{13}$$

For the study case:

$$\dot{\mathcal{L}}\in\left\{\begin{array}{c}\dot{\mathcal{L}}_1\\ \dot{\mathcal{L}}_{1,2}\\ \dot{\mathcal{L}}_2\end{array}\right.\Rightarrow\dot{\varepsilon}^T P\varepsilon+\varepsilon^T P\dot{\varepsilon}\in\left\{\begin{array}{c}\dot{\varepsilon}_1^T P\varepsilon_1+\varepsilon_1^T P\dot{\varepsilon}_1\\ \dot{\varepsilon}_{1,2}^T P\varepsilon_{1,2}+\varepsilon_{1,2}^T P\dot{\varepsilon}_{1,2}\\ \dot{\varepsilon}_2^T P\varepsilon_2+\varepsilon_2^T P\dot{\varepsilon}_2\end{array}\right. \tag{14}$$

For $\dot{L}_i$ for $i = 1$ and 2

$$\dot{\mathcal{L}}_i = \left((\mathbf{f}_i(\mathbf{x},\mathbf{u}) - \mathbf{f}_i(\hat{\mathbf{x}},\mathbf{u})) - \mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} - \mathbf{k}_q\varepsilon\right)^T P\varepsilon + \varepsilon^T P\left((\mathbf{f}_i(\mathbf{x},\mathbf{u}) - \mathbf{f}_i(\hat{\mathbf{x}},\mathbf{u})) - \mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} - \mathbf{k}_q\varepsilon\right) \quad (15)$$

$$\dot{\mathcal{L}}_i = 2\varepsilon^T \mathbf{P}(\mathbf{f}_i(\mathbf{x},\mathbf{u}) - \mathbf{f}_i(\hat{\mathbf{x}},\mathbf{u})) - 2\varepsilon^T \mathbf{P}\left(\mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} + \mathbf{k}_q\varepsilon\right) \quad (16)$$

Here $\mathcal{L}_i$ represents the Lyapunov function for the subsystem i.

Hypothesis 1. The system (10) is Lipchitz bounded:

$$\left\|\varepsilon^T \mathbf{P}(\mathbf{f}_i(\mathbf{x},\mathbf{u}) - \mathbf{f}_i(\hat{\mathbf{x}},\mathbf{u}))\right\| \leq L_i \|\varepsilon\|_P^2 \quad (17)$$

Now,

$$\left\|\varepsilon^T \mathbf{P}\left(\mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} dx + \mathbf{k}_q\varepsilon\right)\right\| \leq \|\varepsilon\|_P \left\|\mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} dx + \mathbf{k}_q\varepsilon\right\| \quad (18)$$

Considering that the nonlinear feedback term is a sigmoid-type function, where *sign* is the well-known *sign* function, the term $\left\|sign * \frac{\varepsilon^n}{1+\varepsilon^n}\right\| \leq 1$ is bounded.

Replacing equations (17) and (18) in equation (16):

$$\dot{\mathcal{L}}_i \leq 2\left[\left(L_i - k_q\right)\|\varepsilon\|_p^2 - k_p\|\varepsilon\|_p\right] \; for\; i = 1\; and\; 2 \quad (19)$$

If $(L_i - k_q) < 0$, choosing $k_q > L_i$ and $k_p > 0$, then $\left(L_i - k_q\right)\|\varepsilon\|_p^2 - k_p\|\varepsilon\|_p \leq 0$ Consequently

$$\dot{\mathcal{L}}_i \leq 0 \quad (20)$$

Now, for the transition region, $\dot{\mathcal{L}}_{1,2}$:

$$\dot{\mathcal{L}}_{1,2} = \left((1-\alpha)(\mathbf{f}_1(\mathbf{x},\mathbf{u}) - \mathbf{f}_1(\hat{\mathbf{x}},\mathbf{u})) - \alpha(\mathbf{f}_2(\mathbf{x},\mathbf{u}) - \mathbf{f}_2(\hat{\mathbf{x}},\mathbf{u})) - \mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} - \mathbf{k}_q\varepsilon\right)^T \mathbf{P}\varepsilon + \varepsilon^T \mathbf{P}\left((1-\alpha)(\mathbf{f}_1(\mathbf{x},\mathbf{u}) - \mathbf{f}_1(\hat{\mathbf{x}},\mathbf{u})) - \alpha(\mathbf{f}_2(\mathbf{x},\mathbf{u}) - \mathbf{f}_2(\hat{\mathbf{x}},\mathbf{u})) - \mathbf{k}_p sign * \frac{\varepsilon^n}{1+\varepsilon^n} - \mathbf{k}_q\varepsilon\right) \quad (21)$$

Then,

$$\dot{\mathcal{L}}_{1,2} = 2\varepsilon^T \mathbf{P}(\alpha(\mathbf{f_1}(\mathbf{x},\mathbf{u}) - \mathbf{f_1}(\hat{\mathbf{x}},\mathbf{u})) - (\mathbf{1}-\alpha)(\mathbf{f_2}(\mathbf{x},\mathbf{u}) - \mathbf{f_2}(\hat{\mathbf{x}},\mathbf{u}))) - \mathbf{2}\varepsilon^{\mathbf{T}}\mathbf{P}\left(\mathbf{k_p sign} * \frac{\varepsilon^{\mathbf{n}}}{\mathbf{1}+\varepsilon^{\mathbf{n}}} + \mathbf{k_q}\varepsilon\right) \quad (22)$$

Under assumption of *Hypothesis 1*,

$$\dot{\mathcal{L}}_{1,2} \leq 2\left[\left(\alpha\mathbf{L}_1 - (1-\alpha)\mathbf{L}_2 - \mathbf{k}_q\right)\|\varepsilon\|_p^2 - \mathbf{k}_p\|\varepsilon\|_p\right] \quad (23)$$

If $\mathbf{k}_q > \alpha\mathbf{L}_1 - (1-\alpha)\mathbf{L}_2$ and $\mathbf{k}_q > 0$

Then

$$\dot{\mathcal{L}}_{1,2} \leq 0 \quad (24)$$

Finally,

$$\dot{\mathcal{L}} \in \begin{cases} 2\left[\left(\mathbf{L}_1 - \mathbf{k}_q\right)\|\varepsilon\|_P^2 - \mathbf{k}_p\|\varepsilon\|_p\right] \leqslant 0, & \hat{\mathbf{x}} \in S_1 \\ 2\left[\left(\alpha\mathbf{L}_1 - (1-\alpha)\mathbf{L}_2 - \mathbf{k}_q\right)\|\varepsilon\|_P^2 - \mathbf{k}_p\|\varepsilon\|_p\right] \leqslant 0, & \hat{\mathbf{x}} \in \Sigma \\ 2\left[\left(\mathbf{L}_2 - \mathbf{k}_q\right)\|\varepsilon\|_P^2 - \mathbf{k}_p\|\varepsilon\|_p\right] \leqslant 0, & \hat{\mathbf{x}} \in S_2 \end{cases} \quad (25)$$

With this, it can be concluded that the estimation error is stable.

4 Numerical results

In order to show the performance of the proposed methodology, numerical simulations were done employing the 23s ode solver library of Matlab?(The MathWorks INC, 2013) in a personal computer. The considered unstructured kinetic models (Haldane-Boulton and Moser-Boulton) were experimentally corroborated (data not shown), where satisfactory correlation coefficients (r^2) were obtained (see Table 3). From the models mentioned above, the switching nonlinear system was developed in accordance with equation (7). The simulations of the system considered the initial conditions indicated in Table 2. The state observer described by equation (11) was implemented with the initial conditions indicated also in Table 2. In order to show the performance of the proposed observed it will be compared to a classical Luenberger observer.

The simulations for the model and observers

are shown in Figures 1 to 3, where the sulfide concentration is considered as the measured output, such that this variable is assumed on-line measured via colorimetric analysis. A local observability analysis, as presented in section 3.1 was done to the switched system's model, being the sulfide concentration the measured output (C=[0,0,1]). The application of the equation (9) provides the global rank of the observability matrix of the considered switched bioreactor model, with $rank[\mathbf{Q_b}] = 7$, concluding in accordance with the Proposition 1, that the switched system is locally partially observable. Although a local observability analysis for each sub-model indicates that the system is observable; the ranks of the observability matrix for the Haldane-Boulton (HB), the (Haldane-Boulton)-(Moser-Boulton) (HB-MB) and Moser-Boulton (MB) kinetic models, are respectively ($rank[Q_{(H-B)}] = 3$; $rank[Q_{(HB-MB)}] = 3$; $rank[Q_{ML}] = 3$).

The estimation performance of the proposed observer for the sulfate and biomass concentration reached the neighborhood of zero for the estimation error (see Figure 4) within few hours, without large overshoots, but with a rather large settling time.

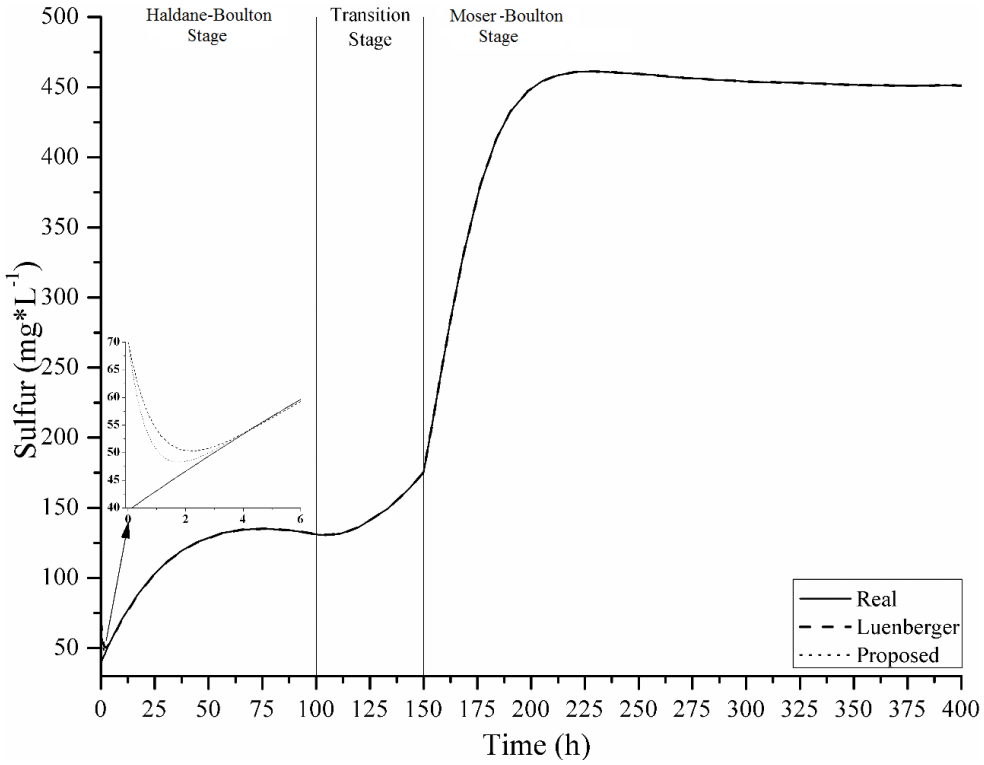

Figure 1. Simulation of the sulfide concentration production (continuous line) and the state observers (Luenberger observer, dashed line and the proposed observer, dotted line).

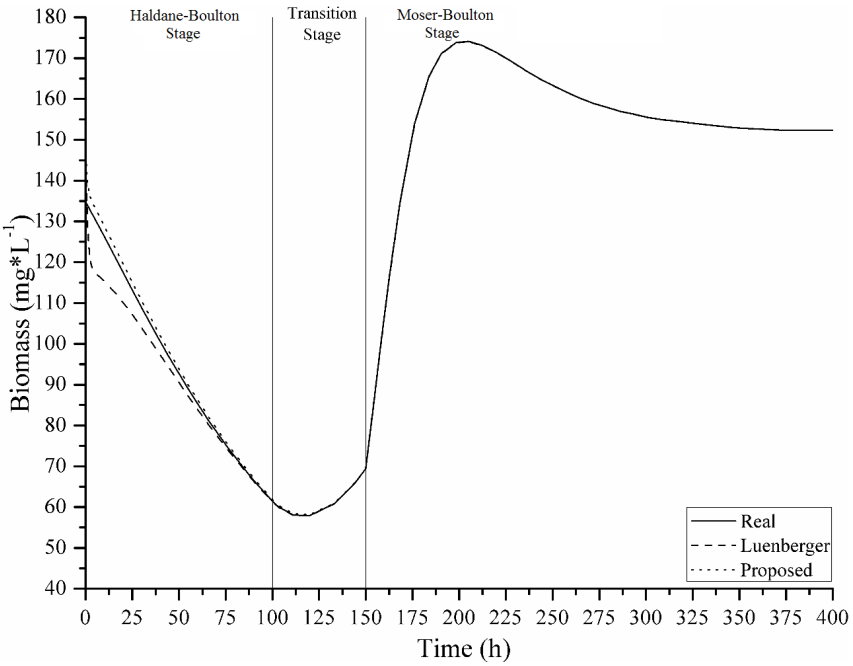

Figure 2. Simulation of the biomass concentration production (continuous line) and the state observers (Luenberger observer, dashed line and the proposed observer, dotted line).

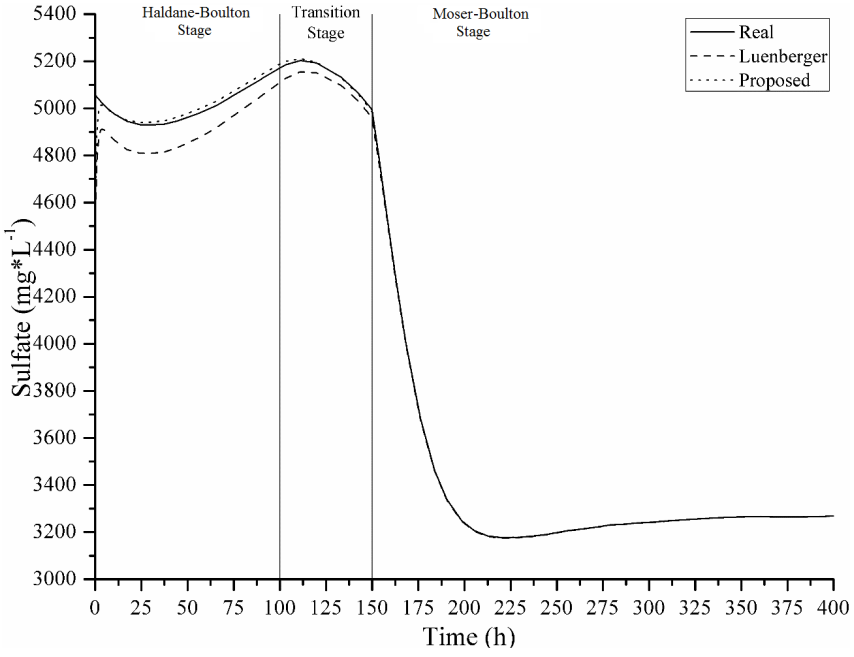

Figure 3. Simulation of the sulfate concentration consumption (continuous line) and the state observers (Luenberger observer, dashed line and the proposed observer, dotted line).

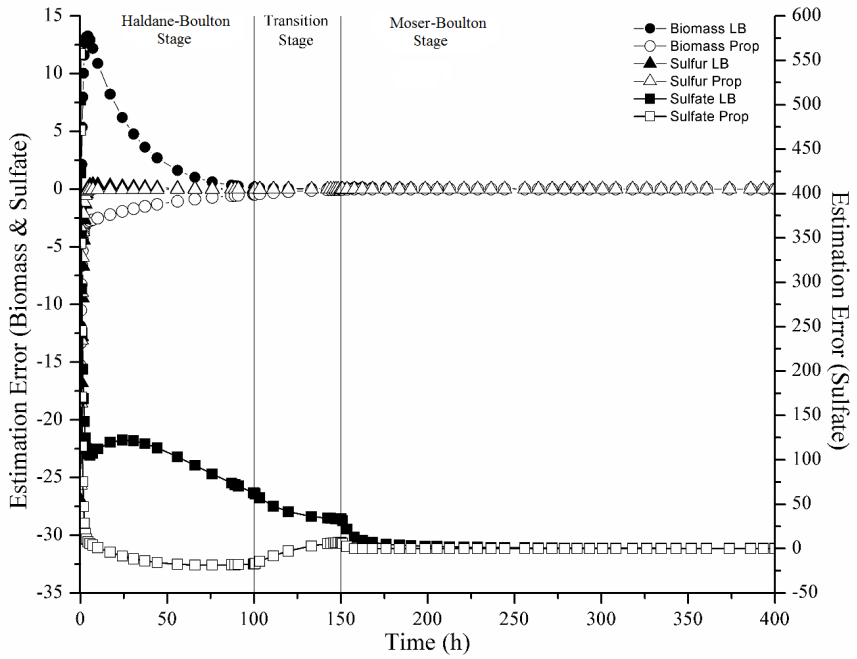

Figure 4. Estimation error for Sulfate (right axis), Biomass and Sulfur (left axis) concentrations for the Luenberger observer (black symbols) and the proposed observer (white symbols).

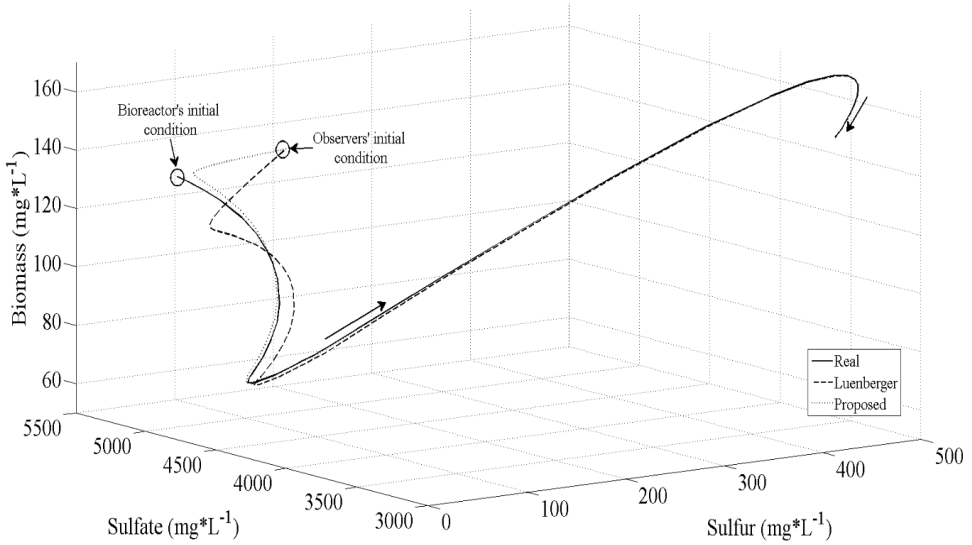

Figure 5. Phase portrait of state variables of the bioreactor (continuous line), the Luenberger (dashed line) and the proposed observers (dotted line).

Table 3. Model Parametric Identification

Kinetic Model	Correlation Coefficient (r^2)			Global (r^2)
	Biomass	Sulfate	Sulfide	
Moser-Boulton	0.9888	0.9550	0.9805	0.9783
Haldane-Boulton	0.9821	0.9765	0.9857	0.9813

Table 4. Initial Conditions

	Sulfate (mg/L)	Biomass (mg/L)	Sulfur (mg/L)
Switched model	5057.47	134.73	39.56
Observers	4500	150	70

A phase portrait representation (Figure 5) of the bioreactor's state variables, when the observers have a different initial condition, is presented. The trajectory of the proposed observer converges faster than the Luenberger's to the real trajectory. Also the proposed observer's trajectory stays close to the real one when the transitions occur.

The state observer described in equation (11) was implemented with the initial conditions shown on Table 4. Where the following linear observer gains (kp) (also the ones used for the Luenbergers observer) were selected heuristically, as:

$$kp \in \begin{cases} kp_1 \\ kp_{1,2} \\ kp_2 \end{cases}$$

Where $k_{p1} = [-15, 1, 1]$; $k_{p1,2} = [50, 1, 1]$ and $k_{p2} = [-100, 1, 1]$. And the observer nonlinear gains (kq) also selected heuristically were:

$$kq \in \begin{cases} kq_1 \\ kq_{1,2} \\ kq_2 \end{cases}$$

Where:$k_{q1} = [5000, 200, 200]$; $k_{q1,2} = [5000, -1, 1]$ and $k_{q2} = [1000, -1, 1]$.

Conclusions

In this work a new class of nonlinear observer is proposed and applied to switching bioreactor model to simulate a kinetic regime change, which is a typical feature in biological systems. The proposed estimation methodology contains a sigmoid-type form of the measured output injection term in order to infer the biomass and sulfate mass concentrations from sulfide concentration measurements in the bioreactor, this sigmoid-type observer is able to reach a satisfactory estimation performance which is better than a standard extended Luenberger-type observer as is shown via numerical simulations. A theoretical framework is provided employing Lyapunov analysis to demonstrate the stability of the estimation error.

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